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SELF-STABILIZING SPANNING TREE ALGORITHM FOR LARGE SCALE SYSTEMS

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Self-Stabilizing Spanning Tree Algorithm for Large Scale Systems *

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Abstract

We introduce a self-stabilizing algorithm that builds and maintains a spanning tree topology on any large scale system. We assume that the existing topology is a complete graph and that nodes may arrive or leave at any time. To cope with the large number of processes of a grid or a peer to peer system, we limit the memory usage of each process to a small constant number of variables, combining this with previous results concerning failure detectors and resource discovery. We provide a formal proof of the algorithm and the results of experiments on a cluster.

Keywords: Distributed Algorithm, Large Scale Systems, Self-Stabilization, Spanning Tree Construction, Failure Detectors.

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1 Introduction

Peer to peer networks and grids are emerging large scale systems that gather thousands of nodes. These networks usually rely on IP to communicate: each node has a unique address used by other nodes to communicate with it.

Classical distributed applications need a notion of neighborhood. Large scale systems shall provide such a notion as a basic system feature to address two issues: provide a list of contacts in the network for each node (their neighbors) and bound the amount of processes with which each node has to communicate. The first issue implies that the virtual topology provided by the system via the neighbor lists should be fully connected, in order to ensure that although only communicating with its neighbors, any node may access any information in the system. The second issue arises because it is not practical to multiplex many communication streams on today's operating systems. As a first attempt to address this issue we choose to build a spanning tree which provides nice properties for efficient diffusion and routing in peer to peer systems.

Since large scale systems are subject to failures that happen mostly because of node departures and link congestion, an algorithm that maintains a topology in such a system needs to tolerate them. A self-stabilizing system is a system that eventually behaves according to its specification whatever the initial configuration is. A set of failures (network failures, topology changes due to process departures or arrivals, or even memory corruptions) leaves the distributed system in a given configuration. Due to its convergence property, the self-stabilizing system will then reach a *legitimate* configuration. From then on, the system conforms to its specification. The idea of self-stabilizing systems was introduced by Dijkstra [4]. The way self-stabilization abstracts any kind of faults by allowing the execution to start from any initial configuration provides a suitable tool to handle the high variety of faults that can happen in large scale systems.

Usually, self-stabilizing algorithms are designed for distributed systems defined by their topology. Each process has a finite set of communication links to exchange messages with its neighbors. In our model, we replace the existence of a complete topology with the notion of neighborhood, based on resource discovery. No process knows the set of its links and, since this set is very large, no process attemps to build it. Moreover, if we compose a higher-level algorithm with ours, it will then benefit from the classical notion of neighborhood, i.e. we provide it as a service.

This model is consistent with most of the Internet peer-to-peer systems, where a process may send messages to another one if and only if it knows its IP address. To discover identifiers, a process may receive them from another process. Of course, every process needs an entry point in the system. In this work we abstract it out using a simple resource discovery service that provides identifiers of other processes to processes that query it.

Since processes can leave the system at any time, it is necessary for the neighbors of a process to be able to decide whether it is still part of the system. Otherwise, the identifiers of crashed processes could not be removed and would prevent the system from converging. Detecting such failures in a purely asynchronous system is impossible [7], so in practice, protocols such as TCP rely on timers, assuming that the Internet is not really asynchronous. In this paper, we use theoretical devices called *failure detectors* [2] to abstract out this partial synchrony: rather than making timing assumptions, we suppose that the system provides a failure detection service.

The issue addressed in this work is the following: we consider a large asynchronous distributed system in which processes communicate by message passing and the capacity of the links is bounded by an unknown constant. Every process p has a unique identifier id_p and may communicate with any other process q, if and only if p knows id_q. Moreover, every process is fitted with two services: a resource discovery service that provides process identifiers, and a failure detection service that provides accurate information on the status of processes from their identifiers. In Section 2, we define formally our computation model. The notion of neighborhood is abstracted out using these two devices. This means that no process ever builds a list of all the identifiers in the system, as it is the case classically, as shown in Section 3. In section 4, we propose a self-stabilizing algorithm in this new model to build a virtual tree, whose degree is bounded by a parameter δ, including all the processes in the system, and we prove its correctness in Section 5. Since we expect our model to bring benefits in terms of scalability, we measure the performances of an experimental implementation and show the results in Section 6. We conclude in Section 7.

2 Model

We denote by $(\mathcal{I}, <)$ the totally ordered finite set of process identifiers in a system and by $P \subseteq \mathcal{I}$ the set of *correct* processes, i.e. those that do not stop (crash) during an execution. We assume the existence of lossless unidirectional FIFO links, each having a capacity bounded by an unknown constant, between each pair of processes. We address the issue of writing an algorithm as if the channels were of unbounded capacity in a system where this is not the case in the same way as Afek and Bremler [1].

2.1 Services

The oracle is a formalized version of the concept of resource discovery, as used in large scale systems. It is intended to replace for the neighbor list used in classical distributed systems. A process executing a guarded rule can

query it, and the answer is an identifier in $\mathcal{I}$. However, no property is guaranteed on the corresponding process; in particular, it could have crashed. Obviously, in order to ensure the connection of the virtual topology, the collection of all the oracles has to satisfy a global property. Formally, if p queries the resource discovery service infinitely many times, it gets the highest identifier of P, and potentially others, infinitely often. It is important to notice that this oracle does not extend the model in any way over the classical asynchronous model: indeed, an oracle that enumerates all the well-formed identifiers in an infinite loop meets the specification.

The failure detector follows the definition given by Chandra and Toueg [2]: a process can query it as part of the execution of a rule, and it returns information on the other processes in the system. This information is generally unreliable, the constraints depend on the *class* of detectors in which the device is. Our model is slightly different from that of Chandra and Toueg since we cannot afford to have a device that returns a list of potentially all the process identifiers in the system due to its large size. Therefore, our detectors provide instead a function $suspect : \mathcal{I} \rightarrow boolean$. This model is equivalent to the original one. To map Chandra and Toueg's model to ours, the suspect predicate can be implemented as follows: true if the process does not belong to the suspect list, false otherwise. Reciprocally, to simulate Chandra and Toueg's model in ours, it is enough to build the list of suspects by applying the suspect predicate to the whole set $\mathcal{I}$.

In this work, all the failure detectors are, according to Chandra and Toueg's nomenclature, in class $\diamond\mathcal{P}$, i.e. eventually perfect detectors. In our model, where all runs are failure-free since all failures are captured in the initial configuration by the self-stabilization model, this class can be defined as follows:

Definition 1. A failure detector is in $\diamond\mathcal{P}$ if and only if after a finite number of queries, its *suspect* function returns *true* if and only if the given identifier is in P and this property remains true from then on.

This is a different formalization of an assumption found in all the related papers. For example, Afek and Bremler [1] suppose that a process knows whether a link is down and Gupta and Srimani [12] use beacon messages to maintain the neighbor lists. This assumption is widely considered necessary, and indeed we prove it in Appendix A.

2.2 Execution

The algorithm is given as a set of guarded rules. Each guard is a boolean expression that can involve the availability of an incoming message, and each rule consumes the message (if any), then can modify the process local state and send messages. We assume a centralized scheduler in the proof for the sake of simplicity. Note that because of the communication model, no

two processes can interfere with each other if we consider a more realistic distributed scheduler, and therefore such an extension is indeed possible. We assume that the scheduler is fair, i.e. any transition that is enabled infinitely many times is eventually triggered.

To account for process identifiers that correspond to stopped (crashed) processes or to no process at all, we adopt the convention that any message sent to a stopped process is lost and that the only entity in the system that may send a message is a correct process.

Definition 2. The *state* of a process is the set of its variables and their values. The *state* of a channel is the ordered list of the messages it contains. A *configuration* is a set $\mathcal{I}$ of process identifiers, a state for each $i \in \mathcal{I}$ and a state for each channel $c_{a \to b} \forall a, b \in \mathcal{I}^2$.

Definition 3. An *execution* is an alternate sequence $C_1, A_1, \ldots, C_i, A_i, \ldots$ such that $\forall i \in \mathbb{N}^*$, applying transition A_i to configuration C_i yields configuration C_{i+1}.

Definition 4. An algorithm is self-stabilizing to $\mathcal{L}$ if and only if (correction) every execution starting from a configuration of $\mathcal{L}$ verifies the specification, (closure) every configuration of all executions starting from a configuration of $\mathcal{L}$ is a configuration of $\mathcal{L}$ and (convergence) starting from any configuration, every execution reaches a configuration of $\mathcal{L}$.

3 Related Works

Besides being interesting in itself, the problem of building a spanning tree in a system is a basic block used in many higher-level algorithms like routing, mutual exclusion, etc. It is therefore particularly relevant to self-stabilizing algorithms since they can be composed, and thus reused.

As an example of an algorithm that relies on this kind of composition, Dolev and Kat [6] designed a distributed self-stabilizing file system that is divided into two components: an algorithm that builds a spanning tree, and a higher-level protocol that makes use of it. The same idea is used by Shen and Tirthapura, who present their routing algorithm for publish-subscribe systems [17] as a protocol that needs to be composed with an existing algorithm that builds a spanning tree. The ability to design algorithms in a modular way is now an acknowledged fundamental feature of self-stabilization.

Classically, self-stabilizing algorithms are most often designed in the classical shared memory model. Dolev, Israeli and Moran [5] studied the differences that have to be taken into account when switching to a message passing paradigm, suitable for describing a system made of geographically distributed machines that exchange messages via communication links, which is currently the only known way to build the large systems we are studying.

Many algorithms were designed to build a spanning tree in the classical shared memory model [11]. In addition to this, they all make at least one hypothesis that we do not want to make, i.e. the existence of a list of neighbors or a predefined topology that is not a complete graph.

Afek and Bremler's *power supply* framework [1] does build a spanning tree in a self-stabilizing fashion in a message-passing model, but it requires every process to know the list of all its outgoing links. It also assumes that the underlying topology is not equivalent to a complete graph. Gupta and Srimani [12] proposed self-stabilizing protocols to build broadcast trees in ad hoc networks, again with the same assumptions.

Garg and Agarwal [10] show another method for building a spanning tree in a message passing model. However, it assumes that the processes are numbered sequentially, which is not the case in practice. We chose the strictly weaker and more realistic hypothesis of only requiring a total order on the process identifiers. Another shortcoming is that part of the algorithm requires all the nodes to send a message to the root, which would lead to link congestion in a large scale system.

Existing peer to peer overlays do need to build trees in a way that is essentially self-stabilizing. The actual structures that are built are usually tailored for their purposes, e.g. a trie [9], a hash table [8], or a Plaxton tree [14]. This is one of the reasons that led us to chose this problem for demonstrating our model.

In order to decide who will be the parent or child of whom, currently implemented algorithms use ad hoc mechanisms, e.g. Dolev and Kat [6] rely on IP multicast and Pastry [15] needs a user-provided metric. In these papers, the resource discovery problem is usually left to the user, or addressed by an ad hoc mechanism such as IP multicasting. By contrast, we define a higher-level mechanism called an *oracle* that provides the calling process with process identifiers. Our goal in doing so is to enable the design of algorithms that do not make explicit use of the low-level characteristics of the underlying network. Furthermore, this allows every process to store only a bounded number of identifiers, thus making our algorithm highly scalable.

To account for the need to know whether a process that has a given identifier is part of the system, we use failure detectors. Introduced by Chandra and Toueg [2], they serve to overcome in a simple and elegant way the impossibility of solving the consensus problem in a purely asynchronous system [7]. They achieve this by allowing processes to access information, possibly wrong, about the liveliness of the others. Their implementation was studied by Chen, Toueg and Aguilera [3]. Interestingly, Chandra and Toueg's view of their failure detectors in practice matches the self-stabilization paradigm: the system behaves according to its specification most of the time and may experience infrequent transient failures. This can be modeled by initializing it arbitrarily and then assuming a failure-free run. This is how failure detectors are used in this paper.

4 Spanning Tree Algorithm

Constants:

– $id_p : id$ unique identifier of process p

– $\delta : \mathbb{N}^*$ bound on the degree of the tree

Variables: (per process p)

– $parent_p : id$ identifier of the parent of p

– $children_p : setofid$ the children of p

Messages:

– $Exists(id)$: sent by root processes to contact new processes to merge with them

– $YouAreMyChild(id)$: sent by processes to accept a new process as its child

– $Neighbor?(id)$: sent by all processes to check consistency of their neighbors

– $NotNeighbor(id)$: negative acknowledge of the $Neighbor?$ request

Procedures and functions:

– $Neighborhood(p : process) : setofid = \{id_q \in children_p \bigcup \{parent_p\}\} \backslash \{id_p\}$

– $Sanity\_check(p : process) : Void =$

SC_1 IF $parent_p < id_p$ THEN $parent_p = id_p$

SC_2 IF $|children_p| > \delta$ THEN $children_p = \emptyset$

SC_3 $children_p = \{id_q \in children_p / id_q < id_p\}$

– $Detect\_failures(p : process) : Void =$

DF_1 IF $parent_p \neq id_p \wedge Suspect(parent_p)$ THEN $parent_p = id_p$

DF_2 $\forall id_q \in children_p$ IF $Suspect(id_q)$ THEN $children_p = children_p \backslash \{id_q\}$

– $RD\_Get()$: id returns an identifier according to the specification of the resource discovery service

– $Suspect(id_p : id) : Bool$ returns true if and only if p is suspect according to the failure detection service

Figure 1: Definitions for the Spanning Tree Algorithm

In this section, we present the self-stabilizing spanning tree algorithm. The tree is distributed among all the processes and described by their *parent* and *children* fields.

We keep the topology free of cycles by means of the following *global invariant*: the identifier of a process must be lower than that of its parent and greater than these of its children. In graph theory, this is known as the *heap invariant*.

Roughly speaking, every process is responsible for checking the consistency of its *neighborhood*, i.e. its parent and its children, using its failure detector to eliminate stopped processes, making sure its parent considers it as a child and vice versa.

In addition, every process that is its own parent, i.e. is root, is responsible for connecting to new processes via the resource discovery service that provides it with identifiers. The root r only sends to the new process p a connection request message ($Exists$) if $p > r$ in order to enforce the global invariant.

The complete algorithm is given in Figures 1 (constants and procedures)

and 2 (guarded rules). Each process has two fields (local variables): *parent*, to store the identifier of its parent in the tree (or its own identifier for a root), and *children*, where it writes the set of its children.

The global invariant is enforced by the *Sanity_check* procedure. Line SC_2 simply ensures that $children_p$ sets initialized with too many components are reset. The other procedure, $Detect\_failures$, checks that the processes in the neighborhood of p are still members of the system. This is the only point where a process sends a query to the failure detection service, thus a process will eventually check the availability of its neighbors only. Both procedures are called at the beginning of each guarded rule. This makes sure that all the rules are executed in a clean environment.

true $\rightarrow$
T_1 $Sanity\_check(p)$
T_2 $Detect\_failures(p)$
T_3 $\forall id_q \in Neighborhood(p)$
SEND $Neighbor?(id_p)$ TO q
T_4 IF $parent_p == id_p$
T_5 THEN *let* $id_q : id_q = RD\_Get()$
T_6 IF $id_q > id_p$
THEN SEND $Exists(id_p)$ TO q

Reception of $Neighbor?(id_q) \rightarrow$
$N?_1$ $Sanity\_check(p)$
$N?_2$ IF $id_p < id_q$
THEN
$N?_3$ IF $parent_p == id_p$ THEN $parent_p = id_q$
$N?_4$ ELSE IF $id_q \notin children_p$ THEN
$N?_5$ IF ($|children_p| < \delta$) $\vee$ ($|children_p| = \delta \wedge$
$\exists id_r \in children_p$ *s.t.* $id_r < id_q$)
$N?_6$ THEN $children_p = (children_p \setminus \{id_r\}) \bigcup \{id_q\}$
$N?_7$ ELSE IF $id_p \neq id_q$ THEN
$N?_8$ SEND $NotNeighbor(id_p)$ TO q

Reception of $NotNeighbor(id_q) \rightarrow$
$\neg N_1$ $Sanity\_check(p)$
$\neg N_2$ IF $parent_p == id_q$ THEN $parent_p = id_p$
$\neg N_3$ $children_p = children_p \setminus \{id_q\}$

Reception of $Exists(id_q) \rightarrow$
E_1 $Sanity\_check(p)$
E_2 IF $id_q < id_p \wedge id_q \notin children_p$ THEN
E_3 IF $|children_p| < \delta$ THEN
E_4 $children_p = children_p \bigcup \{id_q\}$
E_5 SEND $YouAreMyChild(id_p)$ TO q
E_6 ELSE IF $\{id_r \in children_p | id_r > id_q\} \neq \emptyset$
E_7 THEN *let* $id_s \in \{id_r \in children_p / id_r > id_q\}$,
SEND $Exists(id_q)$ TO s
E_8 ELSE
E_9 *let* $id_s \in children_p$,
$children_p = (children_p \setminus \{id_s\}) \bigcup \{id_q\}$
E_{10} SEND $YouAreMyChild(id_p)$ TO q

Reception of $YouAreMyChild(id_q) \rightarrow$
Y_1 $Sanity\_check(p)$
Y_2 IF $parent_p == id_p \wedge id_q > id_p$ THEN $parent_p = id_q$

Figure 2: Guarded Rules for the Spanning Tree Algorithm

There are five guarded rules in the algorithm (Figure 2). The first one is guarded by *true*, which means in practice that it is called regularly by each process. It performs the verifications described above in the neighborhood of the process.

The purpose of the rules that react to $Neighbor?$ and $NotNeighbor$ is to maintain the consistency of the process neighborhood. The $Neighbor?$ message is sent spontaneously, and silently ignored by the receiver if it is in the sender's neighborhood. If it is not, the receiver attemps to add the sender to its neighborhood. If this is impossible, it sends back a negative acknowledgement ($NotNeighbor$) that causes the sender to delete the receiver from its neighborhood.

The rules that handle $Exists$ and $YouAreMyChild$ messages control the merging of trees. Informally, a process p sends $Exists(id_p)$ to q in order to ask q to adopt p. Then q checks whether it should do so (in particular w.r.t. the global invariant), and whether it can (this requires having less than δ children). If q should adopt p but cannot, a finer analysis decides whether q drops a child in favor of p or forwards the $Exists(p)$ message to one of its own children. In any case, q makes sure that p's request is eventually satisfied.

When a process p adds q to its set of children, it sends $YouAreMyChild(p)$ to q. Upon receiving this message, q checks whether it should accept q as its parent and does so if and only if it does not break the global invariant and q is root. This last condition reduces the number of topology changes during the convergence period.

When the system is stabilized, only the process with the highest identifier is a root and there is a single tree. Every process communicates only with its neighbors and the only messages transmitted between two processes are $Neighbor?$ requests. Moreover, only the root continues to query the resource discovery service. If the root receives an identifier, it is lower than its own if it is part of the system, so $Exists(id_{root})$ messages do not circulate. Every process checks, through the failure detection service, the availability of its neighborhood only.

5 Stabilization of the Algorithm

In this section, we prove that our spanning tree algorithm is self-stabilizing to $\mathcal{L}$, defined below.

Definition 5 ($\mathcal{L}$)**.** Let Max be the process with the highest identifier in system S, P the set of processes of S and $\{c_{p\to q}, \forall p, q \in Ps.t. p \neq q\}$ the set of communication channels. Since the set of processes does not change during the executions we consider for the purpose of proving the algorithm, we refer to the processes as $\rho_0 \dots \rho_{|P|-1}$ where $\rho_0 = Max$ and $\forall i \in [1..|P|-1, \rho_i$ is the process with the highest identifier in $P \setminus \{\rho_0 \dots \rho_{i-1}\}$. A configuration C is in $\mathcal{L}$ if and only if, in C,

$$\forall p \in P \begin{cases} p \neq Max \Rightarrow \exists p_1, p_2 \dots p_n \in P \text{ s.t.} & (1) \\ \quad (p = p_1) \wedge (p_n = Max) \bigwedge_{i=1}^{n-1} (parent_{p_i} = id_{p_{i+1}} \wedge id_{p_i} \in children_{p_{i+1}}) & \\ parent_p \geq id_p & (2) \\ children_p = \{q \in P \text{ s.t. } parent_q = id_p\} & (3) \\ |children_p| \leq \delta & (4) \end{cases}$$

Additionally, $\forall p, q \in Ps.t. p \neq q, c_{p\to q}$ may either be empty or, if $q \in neighborhood(p)$, contain any number of $Neighbor?(p)$ messages (5).

Condition (1) implies that there exists a unique path from any process to Max. Conditions (1) and (2) imply that Max is the only root and that any

legitimate configuration satisfies the global invariant. Condition (3) ensures that any process but Max is a child of another process in the system and that only processes of the system are in the tree. Condition (5) implies that no message will break the spanning tree.

The proofs of correction and closure are straightforward. They are given in Appendix B.

The proof of convergence is done in three steps. Firstly, we prove that every execution of the system eventually reaches a configuration from which a few basic properties remain true throughout the execution. Secondly, we define a notion of stable process that formalizes the fact of irrevocably choosing a parent. We prove that a stable process remains so and that every process eventually becomes stable. Thirdly, we prove that a system in which all the processes are stable eventually reaches a legitimate configuration.

Definition 6 (Consistent configuration)**.** Let C be a configuration of system S and p be a process of S, the state of p is consistent in C iff $|children_p| \leq \delta \wedge parent_p \geq id_p \wedge \forall id_q \in children_p, id_q < id_p \wedge \neg suspect(parent_p) \wedge c \in children_p \Rightarrow \neg suspect(c)$.

Similarly, a message in a communication channel is consistent if it results from the complete application of a guarded rule.

Remark 1. Since the global invariant holds for all processes, a consistent configuration does not contain any cycle, i.e. any set of processes $P_1 \dots P_n$ where $\forall i \in [1..n-1], parent_{P_{i+1}} = P_i \wedge parent_{P_1} = P_n$.

It is straightforward to see that from any initial configuration, the system eventually reaches a consistent configuration. Inconsistent states are handled by procedure $Sanity\_Check$, as already discussed. Inconsistent messages eventually reach their destination because all channels are FIFO and the scheduler is fair. The only message that can be forwarded is $Exists$, but only to a child whose identifier is higher than the parameter of the message, thus it can only happen a bounded number of times.

For the purpose of proving the algorithm, we consider an execution starting in a consistent configuration, in which all the failure detectors are converged and no process ever stops (crashes), therefore $suspect(id_p)$ is true if and only if p does not belong to the system. The spontaneous rule calls $Detect\_Failures$, which eliminates such processes, and new identifiers are only written in a process field upon reception of $Exists(id_p)$ or $Neighbor?(id_p)$, where the message was originally sent by p, which is thus alive. Therefore, the $Detect\_Failures$ procedure has no effect during the executions shown below and is not considered. Moreover, the $Sanity\_Check$ procedure does not alter the state of a consistent process. Thus, in what follows, we do not consider the execution of this procedure.

Roughly speaking, in a given configuration, the *stable* processes will not change their parents in the rest of the execution and are connected to the "final" tree.

Definition 7 ($Stable(C)$)**.** Let P be the set of processes of system S, $V = \{c_{p\to q}, (p,q) \in P^2\}$ the set of communication channels between any couple p, q of P^2, and let Max denote the process with the highest identifier. Let C be a configuration of S. The stable processes of C, denoted by $Stable(C)$, is the set of processes such that:

$$p \in Stable(C) \Leftrightarrow$$

$$p \in P \bigwedge \begin{cases} \forall q \in P, id_q > id_p \Rightarrow q \in Stable(C) & (S_{rec}) \\ p \neq Max \Rightarrow \exists p_1, p_2 \dots p_n \in P \text{ s.t.} & \\ \qquad (p = p_1) \wedge (p_n = Max) \bigwedge_{i=1}^{n-1} (parent_{p_i} = id_{p_{i+1}} \wedge id_{p_i} \in children_{p_{i+1}}) & (S_{path}) \\ parent_p \geq id_p & (S_{parent}) \\ \forall q_1, q_2 \in P, Exists(p) \notin c_{q_1 \to q_2} & (S_e) \\ NotNeighbor(id_p) \notin c_{p \to parent_p} \wedge NotNeighbor(id_{parent_p}) \notin c_{parent_p \to p} & (S_{nn}) \end{cases}$$

Condition S_{rec} implies that p is stable only if processes with higher identifiers are also stable. This will lead to a progression of the Stable set from the highest identifiers to the lowest. The S_{path} condition ensures that stable processes are part of the tree and, in conjunction with the S_{parent} condition, that the path to the root is made of stable processes. S_{path} also ensures that the parent of p acknowledges the stability of p, since its parent knows that p is its child and will remain so (since there is no $NotNeighbor$ message between p and its parent according to S_{nn}). Condition S_e also implies that no previous connection request ($Exists()$ messages) originating from a stable process remains in the system. S_e and S_n ensure that no process will reject a child.

We first prove that when a process is stable, it remains so for the rest of the execution. Then, we prove that the set of stable processes eventually grows until all processes are stable.

Stable processes remain stable

Theorem 1. *Let $C_0, A_0, C_1, \dots$ be an execution of the system. Then $Stable(C_0) \subseteq Stable(C_1)$.*

Proof. By induction. Let us first consider a configuration C_0 s.t. $Stable(C_0) = \{Max\}$ and show that Max remains stable in C_1. S_{rec} and S_{path} are true for Max.

S_e is true because the only place in the algorithm where an $Exists$ message is sent out is line T_6, where this is done only to a strictly greater process. S_{nn} is true as well because the only place in the algorithm where a process sends out a $NotNeighbor$ message is sent out is line $N?_8$, where it does not send it to itself. S_{parent} is not broken because in all the places in the algorithm where a value different from id_p is written in $parent_p$, namely $N?_3$ and Y_2, this value cannot be lower than id_p.

Let us now consider a configuration C_0 such that $Stable(C_0) = \rho_0, \ldots, \rho_k$. Our induction hypothesis is: $\rho_0 = Max \ldots \rho_{k-1} \in Stable(C_1)$. We now show that $\rho_k \in Stable(C_1)$.

Since all the higher processes remain stable, S_{rec} still holds. For the same reason, there are only two ways of breaking S_{path}: either p changes the value of $parent_p$, or $parent_p$ deletes p from its set of children.

The former requires the execution by p of one of the following lines: 1) $N?_3$ and Y_2 are not executed because $parent_p \neq id_p$. 2) $\neg N_2$ is not executed because there was no $NotNeighbor$ message in $c_{parent_p \to p}$.

The latter requires the execution by $parent_p$ of one of the following lines: 1) $N?_4$ is not executed because $parent_p$ is stable and thus verifies condition $S_{N?}$: if it receives $Neighbor?$ from a stable process q, then $q \in Neighborhood(p)$. 2) $\neg N_3$ is not executed because there was no $NotNeighbor$ message in $c_{p \to parent_p}$. 3) E_9 is not executed because this requires receiving a message $Exists(id_q)$ s.t. $q > p$, but then q is stable and thus verifies S_e, i.e. there is no such message.

S_{parent} cannot be broken because this would require that p change the value in $parent_p$, which is proven impossible above. S_e is not broken because the only places in the algorithm where an $Exists$ message is sent out are line T_6, where this is done only by roots, and E_7, where a message $Exists(id_q), \forall q$ can only exist in C_1 if there was already such a message in C_0, which is not the case here. S_{nn} is not broken because this would require the execution of line $N?_8$ by either p or $parent_p$, but this is impossible because p and $parent_p$ are in $Stable(C_0)$, moreover $parent_p \in Neighborhood(p)$ and $p \in Neighborhood(parent_p)$.

We conclude that $Stable(C_0) \subseteq Stable(C_1)$. □

Eventually all processes are stable

Let C be a configuration where there is at least one non-stable process and m be the highest non-stable process in C. In this section, we prove that m eventually becomes stable.

Lemma 2. *In an execution starting with C, no process $s \in Stable(C)$ s.t. $|children_s \cap Stable(C)| < \delta$ can send $NotNeighbor(id_s)$ to m.*

Proof. The only place in the algorithm where $NotNeighbor$ messages are produced is upon reception of $Neighbor?$. If s receives this message, then since $id_s > id_m$ by definition of $Stable$ and m, s takes m as a child because it has at least one child lower than id_m and does not produce a $NotNeighbor$ message. □

Lemma 3. *There exists a stable process p such that $|children_p \cap Stable(C)| < \delta$ such that m irrevocably writes id_p in its field $parent_m$ and p irrevocably writes id_m in its field $children_p$.*

Proof. The proof of this technical lemma is given in Appendix C. □

Corollary 4. *m eventually becomes stable.*

Proof. m already satisfies conditions S_{rec}, S_{path}, S_{parent} and S_{nn}. The only line in the algorithm where m could send $Exists(id_m)$ is T_6, but it does not do so because $parent_m \neq id_m$. It is thus enough to show that the remaining $Exists(id_m)$ messages are consumed.

Upon reception of an $Exist(id_m)$ message, a process that is lower than m, i.e. any unstable process, ignores it (line E_2). It it however possible that a stable process s s.t. $id_s \neq parent_m$ add id_m to its set of children and send $YouAreMyChild(id_s)$ to it. Then the following properties apply: 1) Since $parent_m$ is now permanently set to another process (Lemma 3), m ignores this message (line Y_2). 2) s eventually executes its spontaneous rule (by hypothesis on the scheduler) and thus sends $Neighbor?$ to m (line T_3). 3) Upon reception of $Neighbor?(id_s)$, $s \notin Neighborhood(m)$. This is because (a) $id_s > id_m$ and thus $id_s \notin children_m$ and (b) $id_s \neq parent_m$. Therefore, m replies by sending $NotNeighbor(id_m)$ to s (line $N?_8$). 4) Upon reception of $NotNeighbor(id_m)$, s removes id_m from its set of children (line $\neg N_3$). □

Toward a legitimate configuration

Lemma 5. *Let E be an execution of the system S starting from a configuration C s.t. $\forall p \in P$, the state of p is consistent and $p \in Stable(C)$. There exists a configuration L of E such that $L \in \mathcal{L}$.*

The proof consists in showing that the last condition on $\mathcal{L}$ is verified. It is given in Appendix D. We conclude that the algorithm is self-stabilizing to $\mathcal{L}$.

6 Experimental Measurements

We measured the performances of a simple implementation of our algorithm on an experimental cluster platform. It consists in 150 bi-Opteron machines linked by Gigabit Ethernet adapters, part of the Grid Explorer platform. This high-performance cluster allows us to run large scale experiments in a reproducible way. We consider that such an environment is suitable for measuring the performance of a system dedicated to large peer-to-peer environments, since we may carry out large experiments and expect more performance from the network than would be available in an Internet-wide deployment. The details about software are described in Appendix E.

Experimental Method

Measuring the convergence time of self-stabilizing algorithms is not a straightforward process. Indeed, since a self-stabilizing system is distributed, generally no single node has a complete and accurate view of a whole configuration. Since convergence is a predicate on the configurations, determining at which point of the execution the system has converged may imply communications. Such communications could modify the behavior of the system, and may not provide enough accuracy in the measurement.

We instrumented the program implementing the spanning tree protocol with a logging mechanism to record every modification of the state, either due to message receptions or to the execution of the spontaneous transition. This list of states (recorded with the date of the local machine) constitutes the local history of the node. The logging is done in main memory in order to minimize the impact on the behavior of the node compared to disk logging, which requires I/O and much more time. When the user ends the execution, the processes dump their local histories to disk for post-mortem analysis.

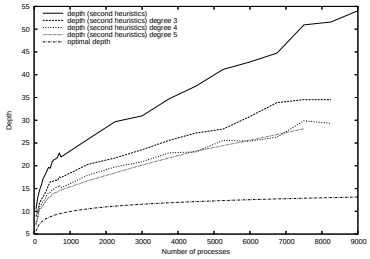
A logical clock mechanism based on Lamport clocks [13] was added to the system, so that it is possible to extract a configuration consistent with the real execution from the set of local histories. Processes measure the physical time since the beginning of the execution using the local physical clock of the machines, which is assumed to have a jitter smaller than 1ms. Processes synchronize the start of their execution using a simple broadcast mechanism.

Running an experiment consists in deploying all the components on the machines, have all of them wait for a broadcast signal, send it, wait for a time long enough to reach a configuration where the expected predicate, e.g. convergence, holds, terminate the experiment, collect the local histories and do a post-mortem analysis of the collection of histories. This analysis consists in extracting the first configuration (according to the Lamport order) where the predicate holds, and returning the maximum value of the physical dates from all the nodes. If the predicate holds in no configuration, the experiment is run again during a longer time. Once an upper bound is determined, we run the experiment 20 times to obtain the mean and standard deviation that are used to plot the curves.

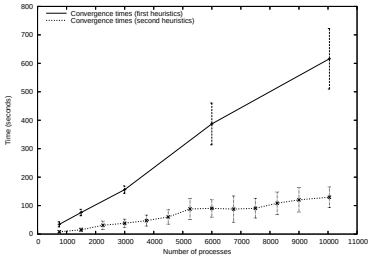
In order to simulate the presence of a high number of machines, we ran several instances of the program on each physical machine. We verified experimentally that this did not saturate the available CPU and network resources.

Results

To measure the scalability of the algorithm, we start it each time from a totally disconnected configuration. This is the worst case: as soon as



(a) Depth of the spanning trees.



(b) Convergence time

Figure 3: Scalability of the spanning tree algorithm.

some processes start earlier than others the convergence time is shorter. We compare two heuristics in the only place in the algorithm left up to the user : the choice of the child that is deleted (line E_9) or to which *Exists* messages are forwarded (line E_7). The first one consists in always selecting the highest identifier, the second one in randomly choosing an eligible child following a homogeneous distribution.

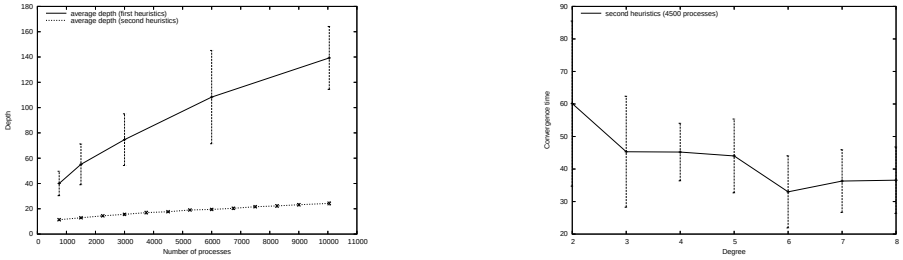
The experiment consisted in building binary trees gathering between 50 and 10050 processes, which meant running 1 to 67 instances of the program on each machine. We present in figure 3 the convergence time and the depth of the tree for 750 to 10050 processes. This figure shows that the convergence phase is divided into two stages: at first the main operation is the insertion of a process in a tree, at this point its depth is optimal, i.e. logarithmic in the number of processes. This is made more efficient by increasing the degree and thus giving each process more children slots. When the main operation becomes tree merging, the depth begins to progress linearly with the number of tree merging, that is linear in the number of nodes. Figure 3(b) shows that the second heuristics yields better performances than the first one. We explain this result below, using other experiments.

Figure 4 displays other characteristics of the algorithm: figure 4(a) reflects the average depth of nodes, an indicator of the quality of the trees that shows that the trees built using the second heuristics have a higher filling rate, due to the random choice for descending *Exists* messages. This is why the second heuristics gives a better convergence time.

Figure 4(b) shows the influence of δ on the convergence time. As expected, one can see that a higher number of children slots allows the logarithmic phase to last longer, thus improving the performances.

7 Conclusion and Future Works

In this work, we address the issue of building virtual topologies for large scale distributed systems, like peer-to-peer or grid systems. In such systems



(a) Average depth of the nodes.

(b) Influence of the degree.

Figure 4: Other characteristics of the spanning tree algorithm.

a node should not have to manage the complete list of participants. For example, it is not desirable to let a node connect to all others because this would be too costly in terms of system resources. We propose to build a bounded degree spanning tree above the virtual complete topology provided by Internet Protocols.

In order to avoid making unnecessary assumptions to solve the spanning tree problem, we consider asynchronous message-passing distributed systems where each node is fitted with the two fundamental services: a resource discovery service and a failure detector. This addresses in a simple way the crucial problems of obtaining an entry point in the system and information about the liveness of other nodes without assuming the presence of user-provided information (entry point) or partial synchrony (failure detection).

The paper presents a self-stabilizing algorithm that uses only $\delta + 1$ process identifiers to build a spanning tree (δ being a bound on the degree of the tree). We present a formal proof of convergence and performance measurements of a prototype implementation of this algorithm and its services for clusters. From a theoretical point of view, the main novelty is that no node of our self-stabilizing algorithm ever knows the list of its neighbors, thus making it highly scalable. On the experimental side, we show that the algorithm performs well enough to argue in favor of the actual application of self-stabilization in practice.

Our intended followup on this work is to design other protocols, building on it, in the same model, so as to explore its viability and efficiency for different problems. We will also study other topologies suitable for large scale systems. It would also be interesting to try to define the notion of stabilisation time in this model. This would require stronger assumptions on the resource discovery service, but which ones exactly is an open question.

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Appendices

A Proof that a failure detector in $\diamond\mathcal{P}$ is necessary

Theorem 6. *A self-stabilizing algorithm that builds a spanning tree in an asynchronous system augmented with failure detectors requires these detectors to be in* $\diamond\mathcal{P}$.

Proof. We use Segall's Propagation of Information with Feedback (PIF) algorithm [16]. It allows any process p to obtain the list of all the processes in the prebuilt spanning tree.

In the terminology of failure detection, a detector D' is *weaker* than a detector D if and only if there exists a *reduction algorithm* that transforms D into D'. Our proof consists in taking any asynchronous distributed system augmented with a failure detector FD in which a spanning tree can be built by a self-stabilizing algorithm and show that it is possible to implement an eventually perfect failure detector from it. As a result, any failure detector in $\diamond\mathcal{P}$ is weaker than FD, which implies that we need an eventually perfect failure detector to solve the spanning tree problem.

Consider a system in which the spanning tree problem is built by an algorithm A. Let AP be the following algorithm: in an infinite loop, execute A then PIF. From AP, let us build a failure detector DAP as follows: when it is queried, DAP gives as its suspect list all the identifiers in P except itself and those it obtained from the last completed call to PIF. When A is stabilized, PIF returns exactly the list of all process in the system and this property remains true. Thus DAP is then a perfect failure detector, i.e. it belongs to class $\diamond\mathcal{P}$. □

B Proofs of Correction and Closure

Lemma 7 (Closure)**.** *Let* $E = C_1, A_1, \ldots, C_i, A_i, \ldots$ *be an execution of the system. If* $C_1 \in \mathcal{L}$ *then* $\forall i \in \mathbb{N}^*$, $C_i \in \mathcal{L}$.

Proof. Let us consider the set of legal actions for a process p:

- Guard *true*.
  - T_1: Sanity_check: the condition in SC_1 is false because of Condition (2) of the definition of $\mathcal{L}$and that of SC_2 is false because of Condition (4). If the condition in SC_3 was true for $id_q \in children_p$ then, by Condition (3), this would mean that $parent_q = id_p$ and thus $parent_q < id_q$, which would break Condition (2). Therefore, nothing happens.
  - T_2: Detect_failures: the condition in DF_1 is false because Condition (3) of the definition of $\mathcal{L}$implies all children of p are in P. The condition in DF_2 is false because if $p \neq Max$, Condition (1)

of the definition of $\mathcal{L}$implies that the parent of p is in P, and if $p = Max$, Condition (2) implies the parent of Max is Max, and thus is in P.

- T_3: $Neighbor?(p)$ messages are sent in communication channels $c_{p \to q}$ for all $q \in neighborhood(p)$, which matches the condition on messages in $\mathcal{L}$.
- T_4: because of Conditions (1) and (2), the condition is only true for Max. However, by definition of Max, there is no $q \in P$ s.t. $id_q > id_{Max}$. Therefore, Max does not send a message to a process in P. It can send an $Exists$ message to a stopped process, but then it is lost and thus does not contradict the definition.

- Reception of a $Neighbor?$ message: for Sanity_check ($N?_1$) see above. The condition in line $N?_2$ cannot be true because of the condition on messages in a legitimate configuration: p can only receive a $Neighbor(q)$ message from a process $q \in Neighborhood(p)$. Notice that consuming the Neighbor? message could not make the resulting configuration illegitimate since it does not break the condition on messages in $\mathcal{L}$.

- All other guards are closed by definition of $\mathcal{L}$.

□

Lemma 8 (Correction)**.** *Let E be an execution of the system starting in configuration C. If $C \in \mathcal{L}$ then E verifies the specification.*

Proof. First, notice that because of closure (Lemma 7), it is enough to prove that any legitimate configuration is correct with respect to the specification.

Conditions (1) and (2) of the definition of $\mathcal{L}$ imply the existence of a unique root, namely Max. Condition (1) implies the existence of a unique path from any node to the root. This means that in any legitimate configuration, the topology described by the processes is a tree. By Condition (4), the degree of this tree is bounded by δ. □

C Proof of Lemma 3

Proof. Suppose $parent_m \notin Stable(C)$. Then, by definition of m, $parent_m < id_m$, and thus the next execution of $Sanity\_Check$ will reset $parent_m$ to id_m. This eventually happens because of the spontaneous rule. Subsequently, m will only accept a parent greater than itself: the check is performed in each place in the algorithm where a write operation is performed on $parent_m$.

Suppose $parent_m = id_m$, i.e. m is root. Then m satisfies the following properties: 1) m executes its spontaneous transition an infinite number of times. This is true by hypothesis on the scheduler. 2) As part of the spontaneous transition, m queries its oracle (line T_5). Since it does so an

infinite number of times, m gets the address of at least one process h, higher than itself ($id_h > id_m$) by definition of the oracle. 3) m sends an $Exists(id_m)$ message to h (line T_6).

Let us now turn to r, the receiver of one of the $Exists(id_m)$ messages that are sent out by m. There are three cases: 1) r has less than δ children. It then adds m to its set of children and sends $YouAreMyChild(id_r)$ to it (line E_4). 2) r has δ children and none of them is greater than m. It then also adds m to its set of children and sends $YouAreMyChild(id_r)$ to it (line E_9). 3) r has δ children and at least one of them, t, is greater than m. This implies that t is stable. Then r forwards the $Exists(id_m)$ message to t (line E_7).

Since the forwarding only takes place downwards in the tree and among stable process, it can only occur a finite number of times. Thus, eventually a stable process u writes id_m in its set of children and sends $YouAreMyChild(id_u)$ to it. Upon reception of this message, since $id_u > id_m$, if $parent_m$ is still id_m then m sets $parent_m$ to id_u (line Y_2).

Now suppose $parent_m \in Stable(C)$ and let us examine the possibilities for m to write another value into this field. There are three places in the algorithm where such a write operation takes place: 1) In the $Sanity\_Check$ procedure, $parent_m$ is erased if it is lower than id_m, which is not the case here. 2) Upon reception of $YouAreMyChild$, a write operation can only take place if $parent_m = id_m$, which is not the case here. 3) Upon reception of $NotNeighbor(id_p)$, if $id_p = parent_m$ then $parent_m$ is reset to id_m (line $\neg N_2$). However, by Lemma 2, no such message is produced. Therefore, this case cannot happen.

Since $parent_m = id_p$ holds for the rest of the execution, $id_m \in children_p$ immediately follows. □

D Towards a legitimate configuration

We prove that a system started in a configuration where all processes are stable eventually reaches a legitimate configuration.

Proof. S_{path} is the same as condition (1) of the definition of $\mathcal{L}$ and S_{parent} is the same as condition (2). Condition (4) is satisfied because all processes are always in a consistent state during E by assumption.

If in C, condition (3) is not satisfied, then $\exists p, q \in P$ s.t. $id_p \in children_q \wedge id_q \neq parent_p$. If $p \notin P$, process q eventually rejects p from its children by executing DF_2. If $p \in P$, process q eventually rejects p from its children: q eventually sends $Neighbor?(id_q)$ to p, and p eventually answers $NotNeighbor(id_p)$ to q (because $id_q \notin children_p$ since q and p are in a consistent state and it is impossible that $id_p < id_q \wedge id_q < id_p$), and then q rejects p from its children. Thus, every process eventually satisfies condition (3). Moreover, if a process satisfies condition (3) in some configuration of

E, then this process satisfies this condition in all subsequent configuration. Indeed, a process p can add a process q to its children iff p receives a message $Exists(id_q)$. But this cannot happen because of condition S_e.

Thus there exists a suffix E' of E s.t. in all configurations of E', conditions (1), (2), (3) and (4) are satisfied.

Conditions S_{nn} and (3) implies that in all configurations of E', we have $\forall p, q \in P$ s.t. $p \neq q, Notneighbor(id_p) \notin c_{p \to q}$. Moreover, condition S_e implies that in all configurations of E', $\forall p, q, r \in P$ s.t. $p \neq q, Exists(id_r) \notin c_{p \to q}$, and so there exists a suffix E'' of E' in which: $\forall p, q, r \in P$ s.t. $p \neq q, YouAreMyChild(id_r) \notin c_{p \to q}$. Thus, condition (5) of the definition of $\mathcal{L}$ is always satisfied in E''. □

E Details on the experiments

E.1 Algorithm

We measured the performances of our algorithm on a straightforward implementation. The only part that requires explanations is the spontaneous rule. Classically, it is triggered by a timeout. We use a simple heuristic to dynamically adapt the duration of this timeout to the activity of the system. It takes five time arguments, namely *initial*, *minimum*, *maximum*, *increment* and *decrement*. At startup, the process triggers the spontaneous rule with a period of *initial* time units. Every time the process changes its state, it subtracts *decrement* from its current timeout, with a lower limit of *minimum*. Every time the application of the spontaneous rule does not change its state, it adds *increment* to its timeout value up to *maximum*. Since the algorithm induces state modifications only when the system has not converged, this heuristic is expected to reduce the time lost in waiting for a message from a process during the convergence phase, and lower the amount of processor and network usage when convergence is achieved.

E.2 Services

In order to run the protocol in a real Internet-based network, one has to implement the two abstractions of which it makes use: the resource discovery service and the failure detector.

We implemented a simple version of a failure detector, as proposed by Chen, Toueg and Aguilera [3]. In their paper, each monitored process knows from the beginning that it should send *heartbeat* messages to its monitor process. The latter considers that the former is alive if it receives a heartbeat message before a given time computed as a function of the networks characteristics, that it permanently keeps estimating, and of the quality of service requested by the user. We adapted this algorithm to our needs by monitoring only the processes that need it and by not suspecting the processes

initially.

Another assumption of the protocol is the existence of a resource discovery service to provide new process identifiers when processes arrive in the system. We assumed that this service is eventually reliable, i.e. that every process which queries the service infinitely often will obtain infinitely often id_{Max}, where Max is the process that has the highest identifier in the system. It may obtain any other identifiers as well.

We designed our resource discovery service to be efficient on a cluster, since this is the experimental testbed we used. Each machine that runs at least one process of the spanning tree algorithm runs a resource discovery daemon. These daemons communicate with each other through multicast channels.

Each resource discovery daemon maintains a bounded list of process identifiers. Every time a process needs to query the resource discovery service, it sends the query (which includes its own identifier) to the daemon running on the same machine. This daemon selects an identifier in its list and returns it to the caller. Regularly, every daemon randomly chooses an identifier in its list and multicasts it to all the other daemons. Every time a daemon receives an identifier (from a process querying for another process, or from the multicast channel), it updates its list using an LRU sorting algorithm.

This algorithm introduces a bias in the answers from the daemon (because of the LRU reordering). Processes that query the daemon are more likely to appear in the answers of the other daemons. This is suitable for our spanning tree protocol, since only the processes that root of their trees request new identifiers. This makes roots likely to obtain each other's identifier and thus facilitates tree merging.