2017-2018

POLYTECH PARIS-SUD

Cycle Ingénieur – 2ème année

Département Informatique

Verification and Validation

Part IV : Proof-based Verification

(II)

Burkhart Wolff
Département Informatique
Université Paris-Sud / Orsay

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Hoare – Logic: A Proof System for Programs

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Logic for Programs ???

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Hoare – Logic: A Proof System for Programs

- Basis: IMP, (following Glenn Wynskell's Book)

  We have the following commands (*cmd*)
  - the empty command SKIP
  - the assignment x:== E (x ∈ V)
  - the sequential compos. c_1 ; c_2
  - the conditional IF cond THEN c_1 ELSE c_2
  - the loop WHILE cond DO c

  where c, c_1, c_2, are cmd's, V variables,
  E an arithmetic expression, cond a boolean expr.

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Hoare Logic vs. Symbolic Execution

- HL is also based notion of a *symbolic state.*

$$state_{sym} = V \rightarrow Set(D)$$

As usual, we denote sets by

$$\{ x \mid E \}$$

where E is a boolean expression.

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Hoare – Logic: A Proof System for Programs

- Core Concept: A Hoare Triple consisting ...
  - of a pre-condition P
  - a post-condition Q
  - and a piece of program cmd

  written:

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P and Q are formulas over the variables V, so they can be seen as set of possible states.

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  $|- \{\sigma::state_{sym} \mid Pre(\sigma(X_1), \ldots, \sigma(X_n)\}$
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  $\{\sigma::state_{sym} \mid Post(\sigma(X_1), \ldots, \sigma(X_n)\}$

  where Pre and Post are sets of states.
  we just write:

  |− {Pre} cmd {Post}

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Hoare Logic vs. Symbolic Execution

- Intuitively:

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Hoare – Logic: A Proof System for Programs

- PL + E + A + Hoare (simplified binding) at a glance:

$$\frac{}{\vdash \{P\}\ \text{SKIP}\ \{P\}} \qquad \frac{}{\vdash \{P[x \mapsto E]\}\ \text{x :== E}\{P\}}$$

$$\frac{\vdash \{P \wedge \mathit{cond}\}\ c\ \{Q\} \qquad \vdash \{P \wedge \neg \mathit{cond}\}\ d\ \{Q\}}{\vdash \{P\}\ \text{IF}\ \mathit{cond}\ \text{THEN}\ c\ \text{ELSE}\ d\{Q\}}$$

$$\frac{\vdash \{P \wedge \mathit{cond}\}\ c\ \{P\}}{\vdash \{P\}\ \text{WHILE}\ \mathit{cond}\ \text{DO}\ c\ \{P \wedge \neg \mathit{cond}\}}$$

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well, states do not change ...

Therefore, valid states remain valid.

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Hoare – Logic: A Proof System for Programs

- The rule for the assignment:

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Example (1):

|−{1≤x ∧ x≤10} x:== x+2 {3≤x ∧ x≤12}

Hoare – Logic: A Proof System for Programs

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Example (2):

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Hoare – Logic: A Proof System for Programs

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essentially case-split.

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Example (3):

$$\vdash \{true\}\ \mathrm{IF}\ 0 \leq x\ \mathrm{THEN\ SKIP\ ELSE}\ x :== -x\ \{0 \leq x\}$$

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Hoare – Logic: A Proof System for Programs

- The rule for the sequence:

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essentially relational composition on state sets.

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

The rule for the sequence.

Example (4):

$$\overline{\vdash \{true\}\ tm :== 1; (sum :== 1; i :== 0)\ \{tm = 1 \wedge sum = 1 \wedge i = 1\}}$$

Hoare – Logic: A Proof System for Programs

The rule for the sequence.

Example (4):

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

The rule for the sequence.

Example (4):

$$\overline{\vdash \{true\}\ tm :== 1; (sum :== 1; i :== 0)\ \{tm = 1 \wedge sum = 1 \wedge i = 1\}}$$

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The rule for the sequence.

Example (4):

$$\dfrac{\dfrac{}{\vdash \{true\} tm :== 1 \{tm = 1\}} \quad \dfrac{\dfrac{}{\vdash \{tm = 1\} sum :== 1 \{B\}} \quad \dfrac{}{\vdash \{B\}\ i :== 0\ \{A\}}}{\vdash \{tm = 1\}\ sum :== 1; i :== 0\ \{A\}}}{\vdash \{true\}\ tm :== 1; (sum :== 1; i :== 0)\ \{tm = 1 \land sum = 1 \land i = 0\}}$$

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Hoare – Logic: A Proof System for Programs

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- The rule for the while-loop.

$$\frac{\vdash \{P \wedge cond\}\ c\ \{P\}}{\vdash \{P\}\ \text{WHILE}\ cond\ \text{DO}\ c\ \{P \wedge \neg cond\}}$$

Critical: The invention of an Invariant P.

If we have an invariant (a predicate that remains stable during loop taversal), then it must be true after the loop. And if states after the loop exist, the negation of the condition must be true.

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- The consequence rule:

$$\frac{P \rightarrow P' \quad \vdash \{P'\}\ cmd\ \{Q'\} \quad Q' \rightarrow Q}{\vdash \{P\}\ cmd\ \{Q\}}$$

Reflects the intuition that P' is a subset of legal states P and Q is a subset of legal states Q'.

The only rule that is not determined by the syntax of the program; it can be applied anywhere in the (Hoare-) proof.

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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$$\frac{true \wedge \neg(0 \leq x) \rightarrow (0 \leq -x) \quad \overline{\vdash \{(0 \leq x)[x \mapsto -x]\}\ x :== -x\ \{0 \leq x\}} \quad 0 \leq x \rightarrow 0 \leq x}{\vdash \{true \wedge \neg(0 \leq x)\}\ x :== -x\ \{0 \leq x\}}$$

Hoare – Logic: A Proof System for Programs
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Hoare – Logic: A Proof System for Programs
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Hoare – Logic: A Proof System for Programs
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Hoare – Logic: A Proof System for Programs

- A handy derived rule (False):

$$\frac{}{\vdash \{false\}\ cmd\ \{false\}}$$

Proof: by induction over cmd !

A very handy corollary of this and the consequence is rule (FalseE):

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Hoare – Logic: A Proof System for Programs

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- Another handy corollary of (False):

$$\frac{}{\vdash \{P \wedge \neg cond\}\ \mathrm{WHILE}\ cond\ \mathrm{DO}\ c\ \{P \wedge \neg cond\}}$$

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by consequence, while-rule,
P and cond-contradiction,
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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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- Yet another handy corollary of (consequence):

$$\frac{P = P' \quad \vdash \{P'\}\ cmd\ \{Q'\} \quad Q' = Q}{\vdash \{P\}\ cmd\ \{Q\}}$$

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by consequence and the fact that $P = P'$ infers
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Note: We will apply this rule implicitly, allowing local massage of pre- and postconditions.

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

- Example (6):

$$\frac{}{\vdash \{true\}\ \text{WHILE}\ true\ \text{DO}\ SKIP\ \{x = 42\}}$$

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Note:

Hoare-Logic is a calculus for partial correctness; on non-terminating programs, it is possible to prove *anything*!

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

- Example (7):

$$\frac{}{\vdash \{true\}\ \text{WHILE}\ x < 2\ \text{DO}\ x :== x + 1\ \{2 \leq x\}}$$

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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where $I'' = I'[x \mapsto x{+}1]$ and where we need solutions to:

$A = true \rightarrow I$

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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    e.g. $x \leq 2$ or $x \leq 5$ do the trick !

Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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  Remarks:
  - This proof rises the idea of particular construction method of Hoare-Proofs, which can be automated:
    - apply the consequence rule only at entry points of (the body of) loops (deterministic!)
    - extract the implications used in these consequence rule
    - try to find solutions for these implications (worst case: ask the user ...)
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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: A Proof System for Programs

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Hoare – Logic: Summary

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- Note: Validity is a « partial correctness notion »

  proof under condition that the program terminates. For non-terminating programs, the calculus allows to prove anything

- The Proof-Method is therefore two-staged:
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Theorem: Correctness of the Hoare-Calculus

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Formal Proof

- Can be very hard – up to infeasible (no one will probably ever prove correctness of MS Word!)
- Proof Work typically exceeds Programming work by a factor 10!
- Tools and Tool-Chains necessary
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  Well, yes, practically.
  (See Hales Article in AMS: "Formal Proof", 2008.
  http://www.ams.org/ams/press/hales-nots-dec08.html)

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Verification : Test or Proof

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- Requires Testability of Programs (initialitzable, reproducible behaviour, sufficient control over non-determinism)
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- Requires a Formal Specification
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Test and Proof are Complementary ...

- ... and extreme ends of a continuum : from static analysis to formal proof of "deep system properties"
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