

*L3 Mention Informatique  
Parcours Informatique et MIAGE*

# Génie Logiciel Avancé - Advanced Software Engineering

## Annotating UML with MOAL

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# Plan of the Chapter

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- ❑ Syntax & Semantics of our own language

## MOAL

- mathematical
- object-oriented
- UML-annotation
- language

(conceived as the „essence“ of annotation  
languages like OCL, JML, Spec#, ACSL, ...)

# Plan of the Chapter

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- Concepts of MOAL
  - Basis: Logic and Set-theory
  - MOAL is a Typed Language
  - Basic Types, Sets, Pairs and Lists
  - Object Types from UML
  - Navigation along UML attributes and associations
- (Idea from OCL and JML)
- Purpose :
  - Class Invariants
  - Method Contracts with Pre- and Post-Conditions
  - Annotated Sequence Diagrams for Scenarios, . . .

# Plan of the Chapter

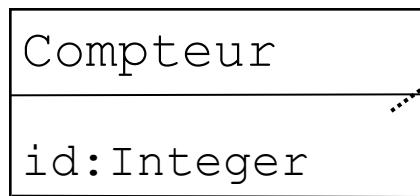
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- ❑ Ultimate Goal:  
Specify system components to improve analysis, design, test and verification activities
- ❑ . . . understanding how some analysis tools work . . .
- ❑ . . . understanding key concepts such as class invariants and contracts for analysis and design

# Motivation: Why Logical Annotations

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- More precision needed  
(like JML, VCC) that constrains an underlying **state  $\sigma$**

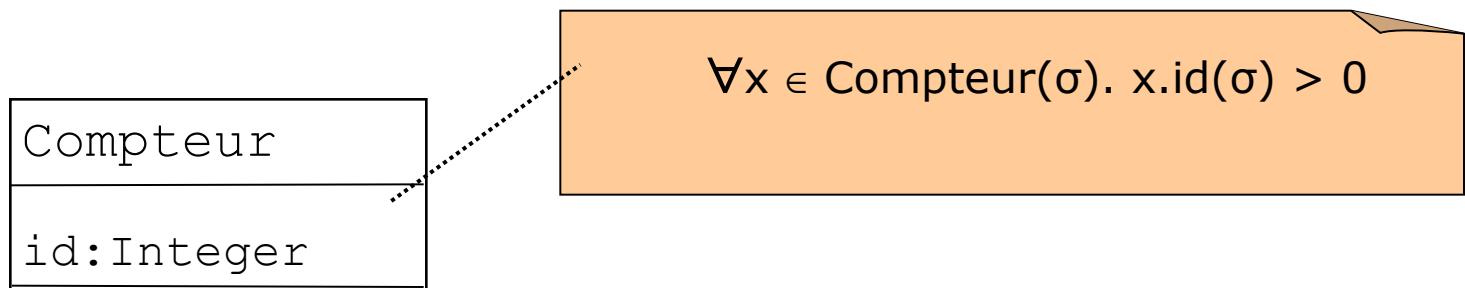


$x.\text{id}$  must be larger 0  
(for any object  $x$  of Class Compteur)

# Motivation: Why Logical Annotations

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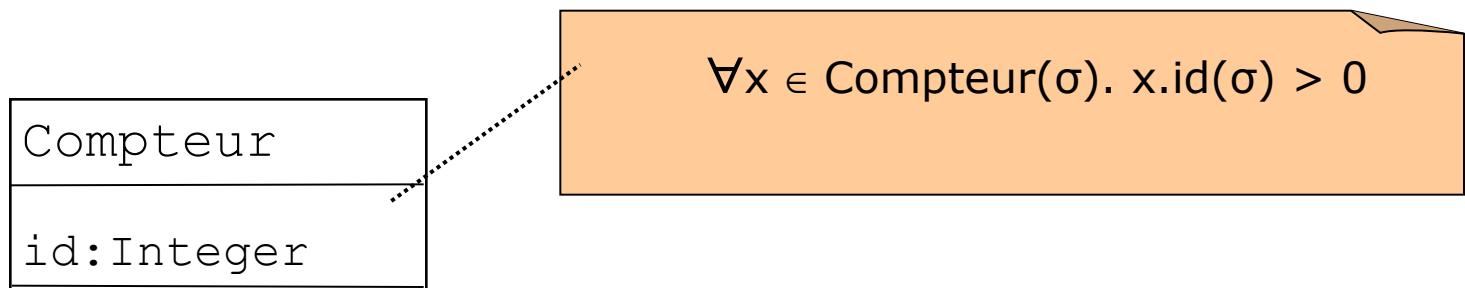
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# Motivation: Why Logical Annotations

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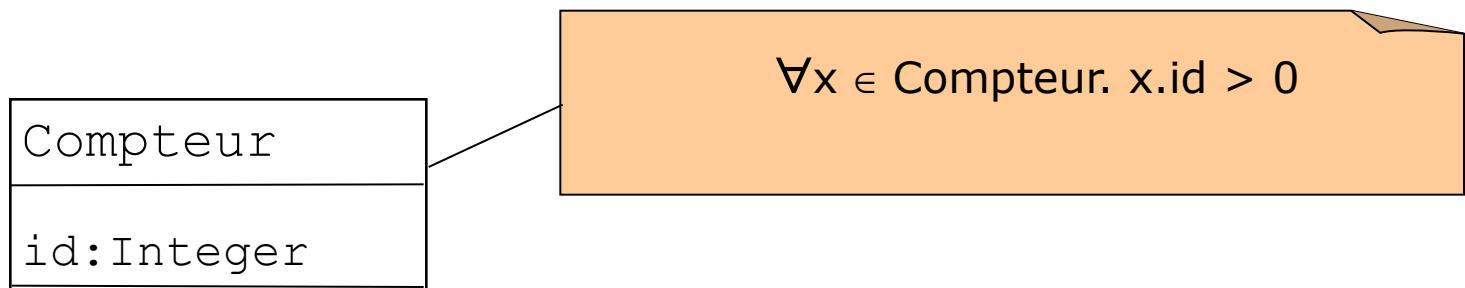
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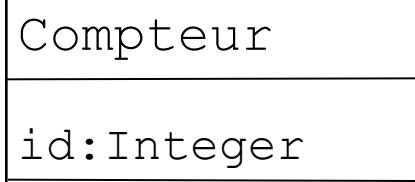


... by abbreviation convention if no confusion arises.

# Motivation: Why Logical Annotations

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- More precision needed  
(like JML, VCC) that constrains an underlying **state  $\sigma$**



definition  $\text{inv}_{\text{Compteur}}(\sigma) \equiv \forall x \in \text{Compteur}(\sigma). x.\text{id}(\sigma) > 0$

... or by convention

definition  $\text{inv}_{\text{Compteur}} \equiv \forall x \in \text{Compteur}. x.\text{id} > 0$

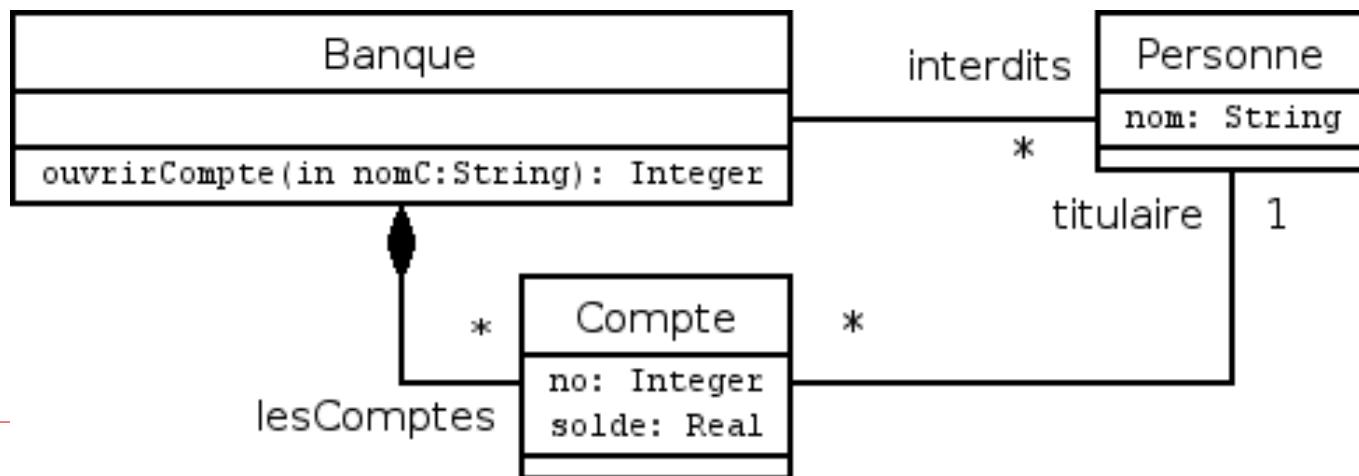
... or as mathematical definition in a separate  
document or text ...

# A first Glance to an Example: Bank

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Opening a bank account. Constraints:

- ❑ there is a blacklist
- ❑ no more overdraft than 200 EUR
- ❑ there is a present of 15 euros in the initial account
- ❑ account numbers must be distinct.



# A first Glance to an Example: Bank (2)

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```
definition unique ≡ isUnique(.no) (Compte)

definition noOverdraft ≡  $\forall c \in \text{Compte}. c.\text{id} \geq -200$ 

definition preouvrirCompte(b:Banque, nomC:String) ≡
     $\forall p \in \text{Personne}. p.\text{nom} \neq \text{nomC}$ 

definition postouvrirCompte(b:Banque, nomC:String, r::Integer) ≡
    | {p ∈ Personne | p.nom = nomC ∧ isNew(p)} | = 1
    ∧ | {c ∈ Compte | c.titulaire.nom = nomC} | = 1
    ∧  $\forall c \in \text{Compte}. c.titulaire.nom = \text{nomC} \rightarrow c.\text{solde} = 15$ 
        ∧ isNew(c)
```

# MOAL: a specification language?

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- In the following, we will discuss the

MOAL Language in more detail ...

# Syntax and Semantics of MOAL

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- The usual logical language:

- True, False
- negation :  $\neg E$ ,
- or:  $E \vee E'$ , and:  $E \wedge E'$ , implies:  $E \rightarrow E'$
- $E = E'$ ,  $E \neq E'$ ,
- if  $C$  then  $E$  else  $E'$  endif
- let  $x = E$  in  $E'$
  
- Quantifiers on sets and lists:

$$\forall x \in \text{Set}. \ P(x)$$
$$\exists x \in \text{Set}. \ P(x)$$

# Syntax and Semantics of MOAL

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- MOAL is (like OCL or JML) a typed language.
  - Basic Types:  
Boolean, Integer, Real, String
  - Pairs:  $X \times Y$
  - Lists: List( $X$ )
  - Sets: Set( $X$ )

# Syntax and Semantics of MOAL

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- The arithmetic core language.  
expressions of type Integer or Real :

- 1, 2, 3 ... resp. 1.0, 2.3, pi.
- - E, E + E',
- E \* E', E / E',
- abs(E), E div E', E mod E' ...

# Syntax and Semantics of MOAL

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- The expressions of type String:

- $S \text{ concat } S'$
- $\text{size}(S)$
- $\text{substring}(i, j, S)$
- 'Hello'

# Syntax and Semantics of MOAL Sets

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- $|S|$  size as Integer
- $\text{isUnique}(f)(S) \equiv \forall x, y \in S. f(x) = f(y) \rightarrow x = y$
- $\{\}, \{a, b, c\}$  empty and finite sets
- $e \in S, e \notin S$  is element, not element
- $S \subseteq S'$  is subset
- $\{x \in S \mid P(x)\}$  filter
- $S \cup S', S \cap S'$  union , intersect  
between sets of same type
  
- Integer, Real, String ...  
are symbols for the set  
of all Integers, Reals, ...

# Syntax and Semantics of MOAL Pairs

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- $(X, Y)$  pairing
- $\text{fst}(X, Y) = X$  projection
- $\text{snd}(X, Y) = Y$  projection

# Syntax and Semantics of MOAL Lists

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Lists  $S$  have the following operations:

- $x \in L$  -- is element (overload!)
- $|S|$  -- length as Integer
- $\text{head}(L), \text{last}(L)$
- $\text{nth}(L, i)$  -- for  $i$  between 0 et  $|S|-1$
- $L @ L'$  -- concatenate
- $e \# S$  -- append at the beginning
- $\forall x \in \text{List}. P(x)$  -- quantifiers :
- $[x \in L \mid P(x)]$  -- filter
- Finally, denotations of lists: [1,2,3], ...

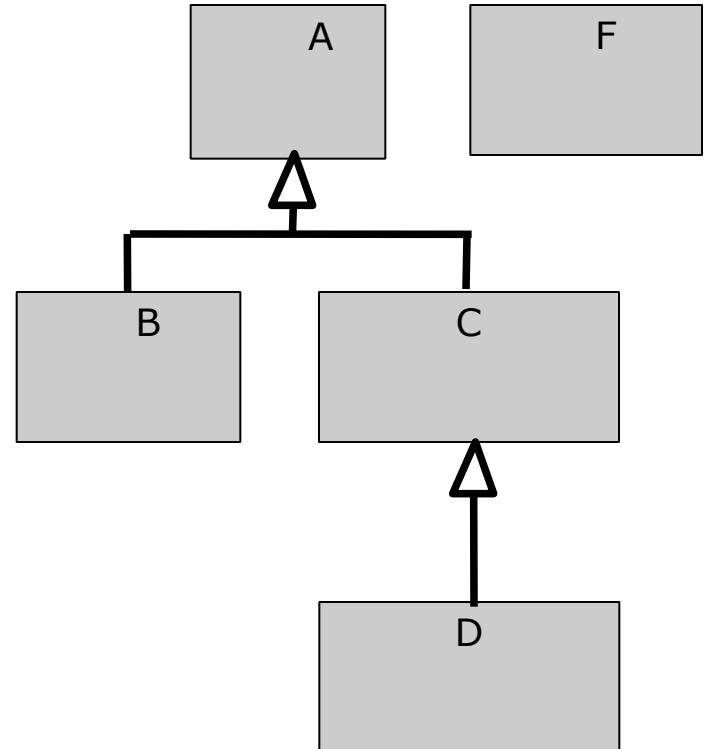
# Syntax and Semantics of Objects

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- Objects and Classes follow the semantics of UML
  - inheritance / subtyping
  - casting
  - objects have an id
  - NULL is a possible value in each class-type
  - for any class A, we assume a function:

$$A(\sigma)$$

which returns the set of objects of class A in state  $\sigma$  (the « instances » in  $\sigma$ ).

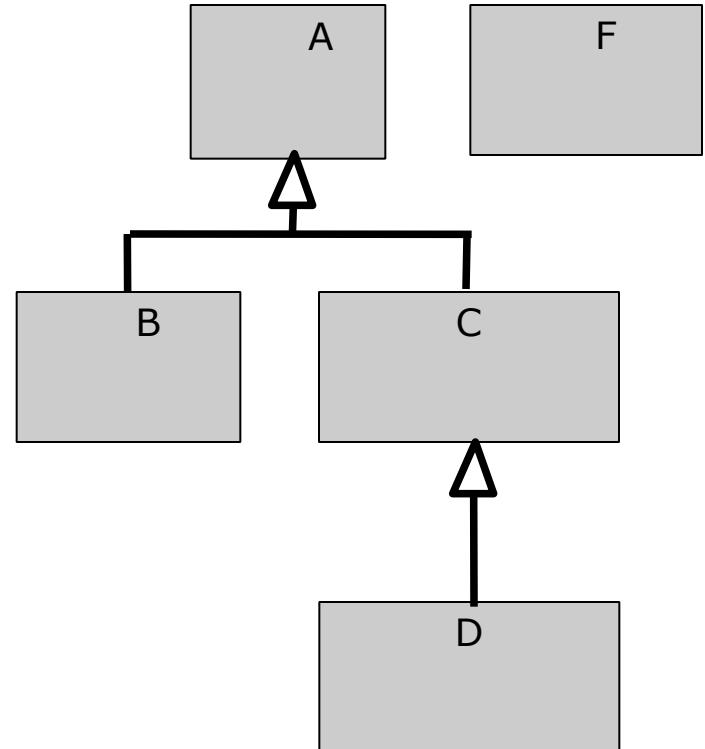


# Syntax and Semantics of Objects

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- Objects and Classes follow the semantics of UML

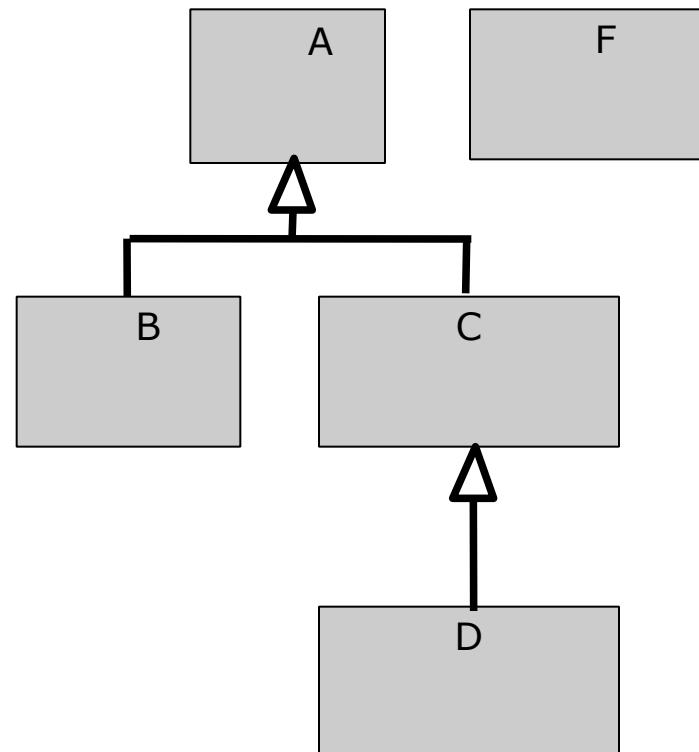
Recall that we will drop the index ( $\sigma$ ) whenever it is clear from the context



# Syntax and Semantics of Objects

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- ❑ As in all typed object-oriented languages casting allows for converting objects.
- ❑ Objects have two types:
  - the « apparent type »  
(also called static type)
  - the « actual type »  
(the type in which an  
object was created)
  - casting changes the apparent type  
along the class hierarchy, but  
not the actual type



# Syntax and Semantics of Objects

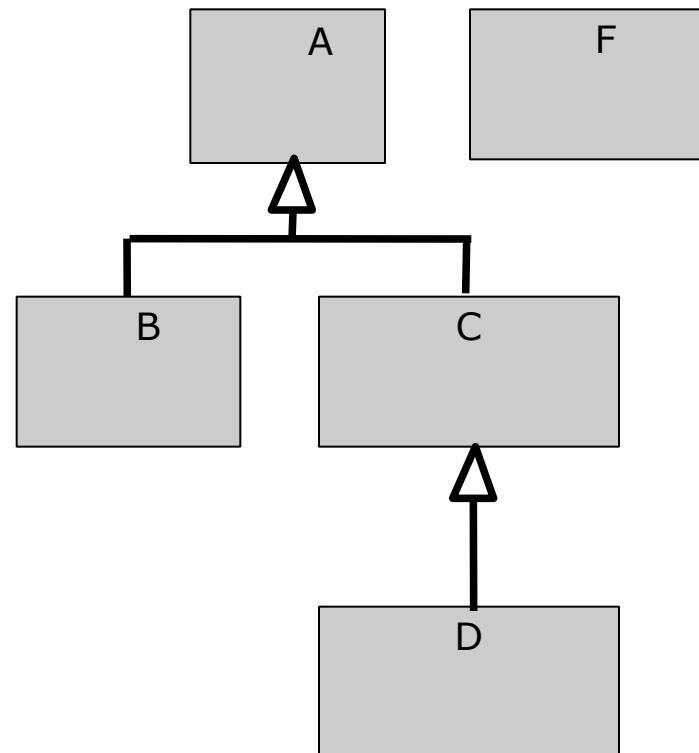
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- Assume the creation of objects
  - a in class A, b in class B,
  - c in class C, d in class D,
- Then casting:

$\langle F \rangle b$  is illtyped

$\langle A \rangle b$  has apparent type A,  
but actual type B

$\langle A \rangle d$  has apparent type A,  
but actual type D



# Syntax and Semantics of OCL / UML

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- >We will also apply cast-operators to an entire set: So

$\langle A \rangle_B(\sigma)$  (or just:  $\langle A \rangle_B$ )  
is the set of instances  
of  $B$  casted to  $A$ .

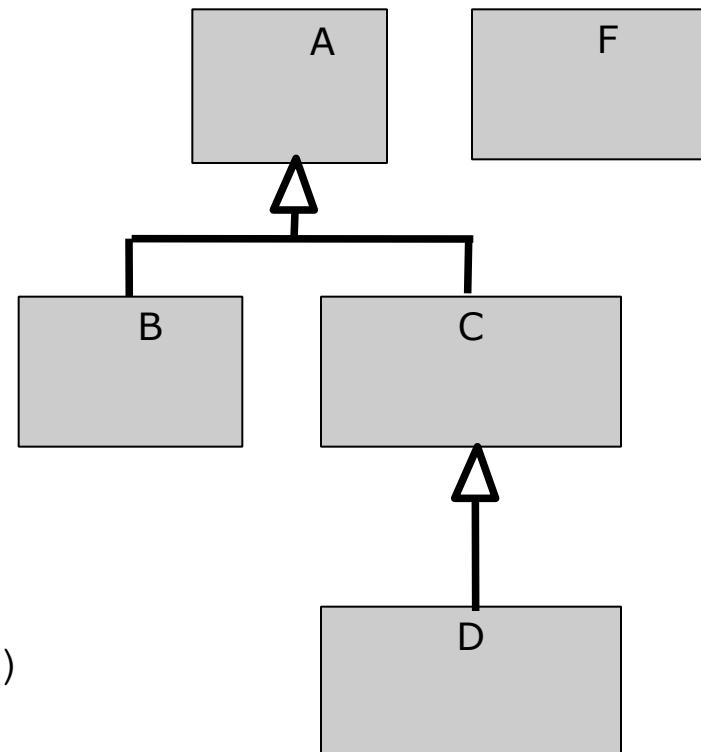
We have:

$$\langle A \rangle_B \cup \langle A \rangle_C \subseteq A$$

but:

$$\langle A \rangle_B \cap \langle A \rangle_C = \{ \}$$

and also:  $\langle A \rangle_D \subseteq A$  (for all  $\sigma$ )



# Syntax and Semantics of Objects

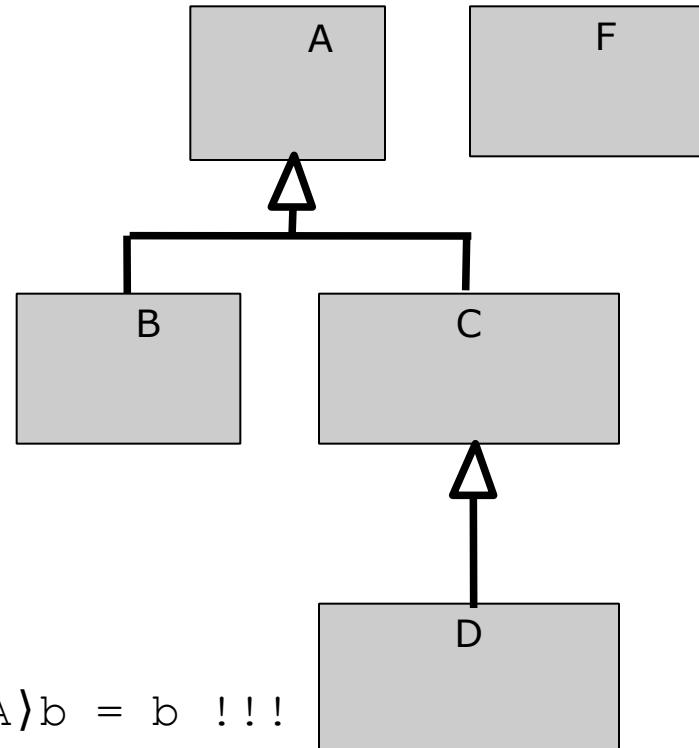
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- Instance sets can be used to determine the actual type of an object:

$x \in B$

corresponds to Java's instanceof or OCL's isKindOf. Note that casting does NOT change the actual type:

$\langle A \rangle b \in B, \text{ and } \langle B \rangle \langle A \rangle b = b !!!$



# Syntax and Semantics of Objects

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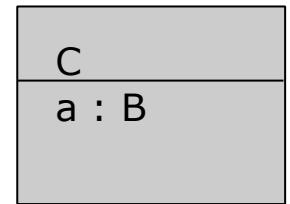
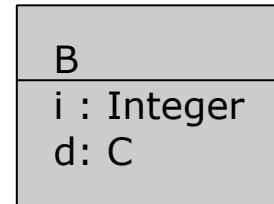
- ❑ Summary:
  - there is the concept of **actual** and **apparent type** (anywhere outside of Java: **dynamic** and **static type**)
  - type tests check the former
  - type casts influence the latter,  
but not the former
  - up-casts possible
  - down-casts invalid
  - consequence:  
up-down casts are identities.

# Syntax and Semantics of Object Attributes

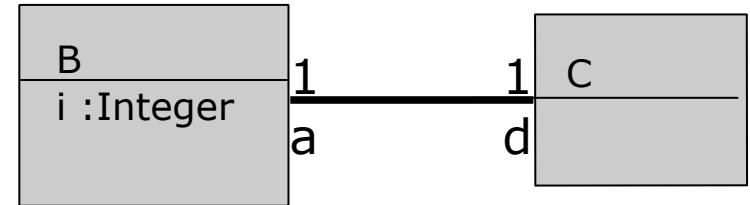
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- Objects represent structured, typed memory in a state  $\sigma$ . They have attributes.

They can have class types.



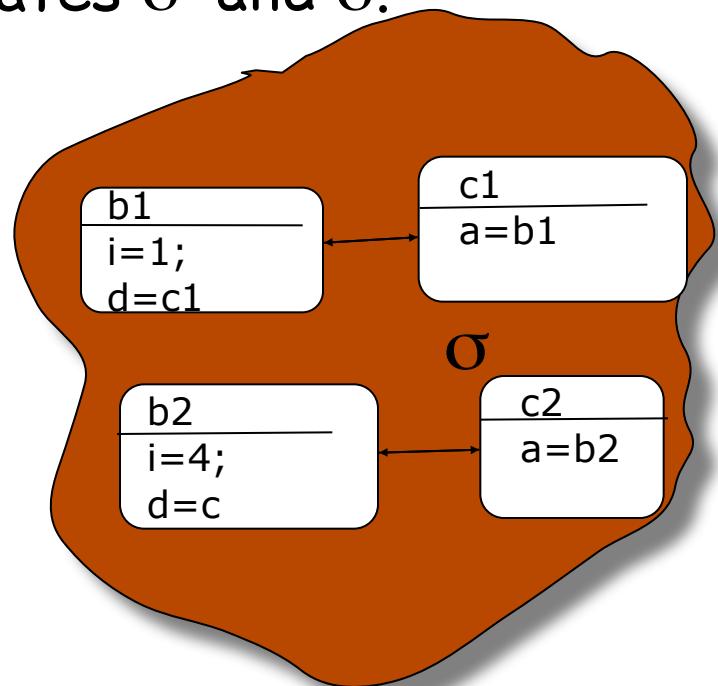
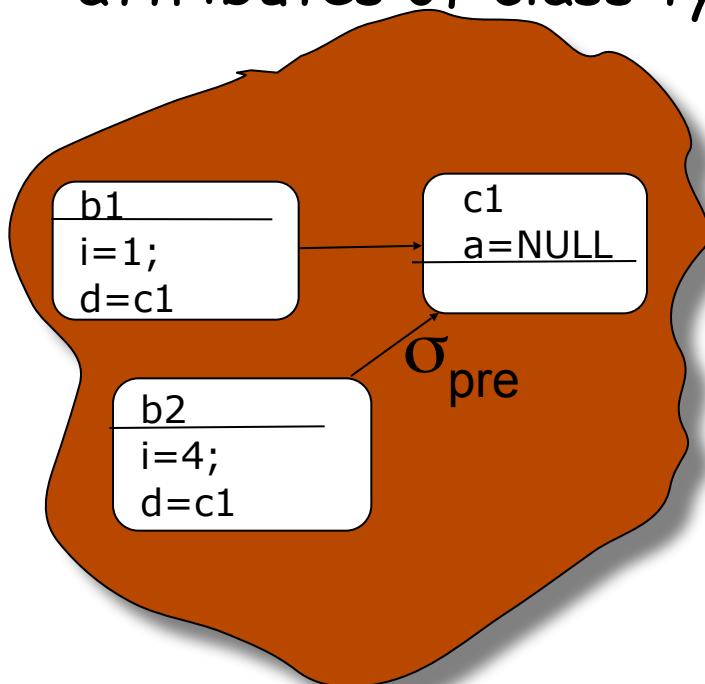
- Reminder: In class diagrams, this situation is represented traditionally by Aggregations (somewhat sloppily: Associations)



# Syntax and Semantics of Object Attributes

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- Example:  
attributes of class type in states  $\sigma'$  and  $\sigma$ .



# Syntax and Semantics of Object Attributes

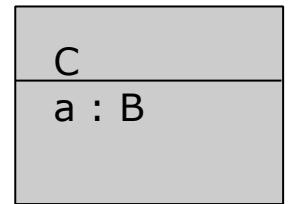
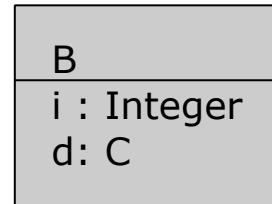
---

- each attribute is represented by an *accessor-function* in MOAL. The class diagram right corresponds to the declaration of them:

.i( $\sigma$ ) :: B -> Integer

.a( $\sigma$ ) :: C -> B

.d( $\sigma$ ) :: B -> C



- This makes navigation expressions possible:

➤ b1.d( $\sigma$ ) :: C  
c1.a( $\sigma$ ) :: B

b1.d( $\sigma$ ).a( $\sigma$ ).d( $\sigma$ ).a( $\sigma$ ) . . .

# Syntax and Semantics of Object Attributes

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- each attribute is represented by a function in MOAL.

The class diagram right corresponds to declaration of accessor functions:

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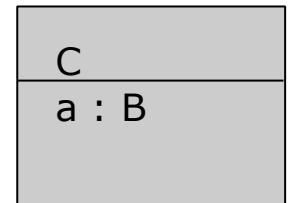
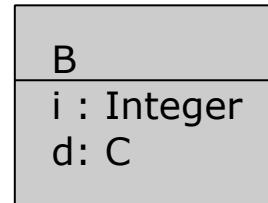
.a( $\sigma$ ) :: C -> B

.d( $\sigma$ ) :: B -> C

- Applying the  $\sigma$ -convention, this makes the following navigation expression syntax possible:

➤ b1.d :: C  
c1.a :: B

b1.d.a.d.a ...



# Syntax and Semantics of Object Attributes

---

- ❑ Assessor functions “dereferentiate” pointers in a given state
  - ❑ Assessor functions of class type are **strict** wrt. NULL.
    - $\text{NULL.d} = \text{NULL}$
    - $\text{NULL.a} = \text{NULL}$
    - Note that navigation expressions depend on their underlying state:  
 $b1.d(\sigma_{\text{pre}}) . a(\sigma_{\text{pre}}) . d(\sigma_{\text{pre}}) . a(\sigma_{\text{pre}}) = \text{NULL}$   
 $b1.d(\sigma) . a(\sigma) . d(\sigma) . a(\sigma) = b1 \quad !!!$
- (cf. Object Diagram pp 28)

# Syntax and Semantics of Object Attributes

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- ❑ Assessor functions of class type are  
**strict** wrt. NULL.
  - $\text{NULL.d} = \text{NULL}$
  - $\text{NULL.a} = \text{NULL}$
  - The  $\sigma$  convention allows to write :

$\text{old}(b1.d.a.d.a) = \text{NULL}$

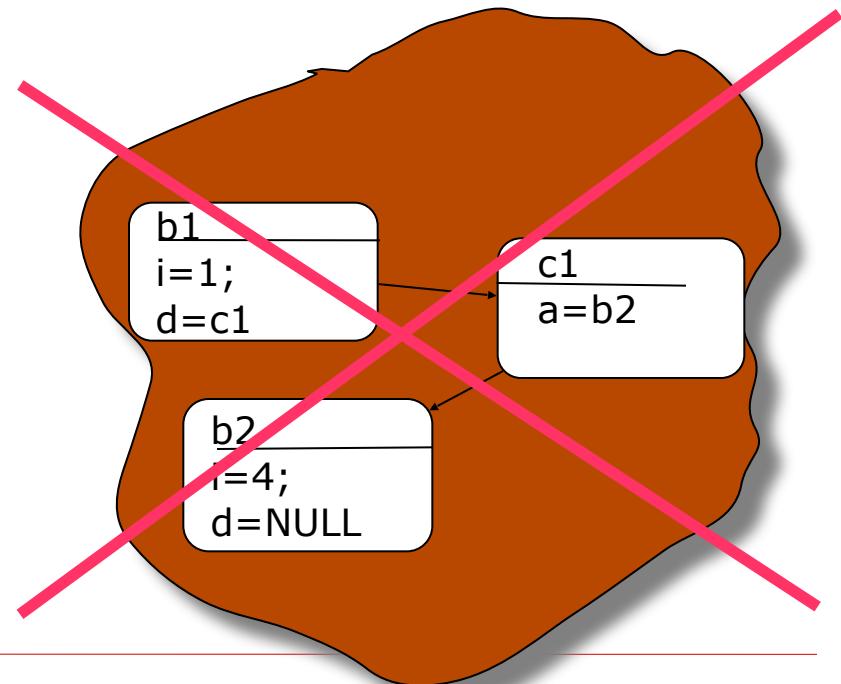
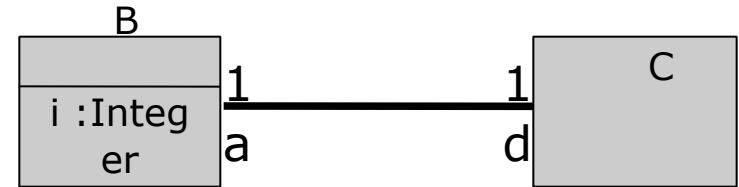
$b1.d.a.d.a = b1$  !!!

(cf. Object Diagram pp 28)

# Syntax and Semantics of Object Attributes

- ❑ Note that associations are meant to be « relations » in the mathematical sense.  
(Here, we treat them like aggregations, which is strictly speaking a design step)

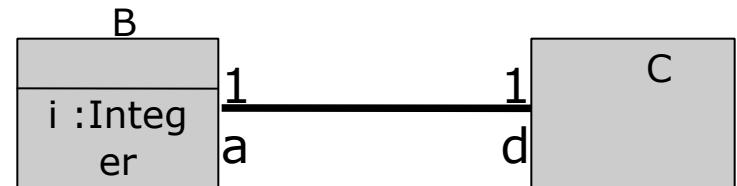
Thus, states (object-graphs) of this form do not represent an association of the cardinality 1 - 1:



# Syntax and Semantics of Object Attributes

- This is reflected by 2 « association integrity constraints ».

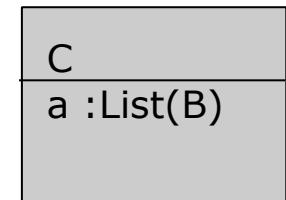
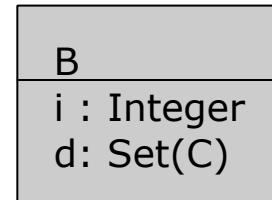
For the 1-1-case, they are:



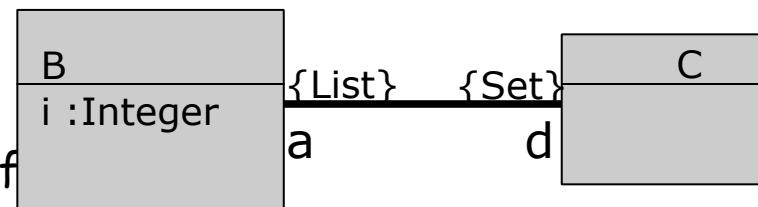
- definition  $\text{ass}_{B.d.a} \equiv \forall x \in B. x.d.a = x$
- definition  $\text{ass}_{C.a.d} \equiv \forall x \in C. x.a.d = x$

# Syntax and Semantics of Object Attributes

- Attributes can be Lists or Sets of class types:



- Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)

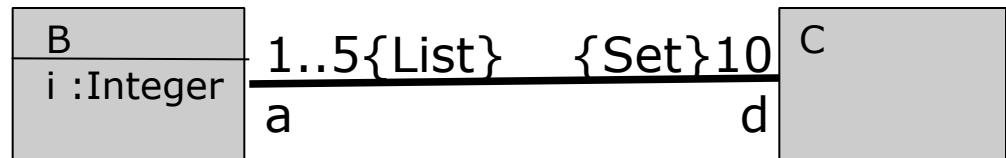


- In analysis-level Class Diagrams, the type information is still omitted; due to overloading of  $\forall x \in X. P(x)$  etc. this will not hamper us to specify ...

# Syntax and Semantics of Object Attributes

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- Cardinalities in Associations can be translated canonically into MOCL invariants:

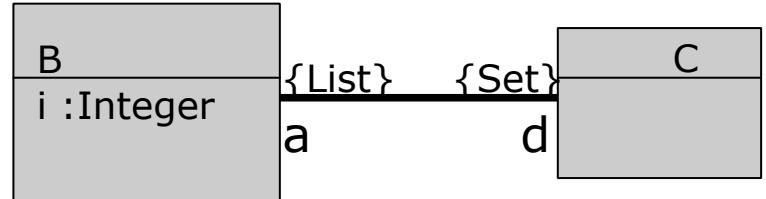


- definition  $\text{card}_{B.d} \equiv \forall x \in B. |x.d| = 10$
- definition  $\text{card}_{C.a} \equiv \forall x \in C. 1 \leq |x.a| \leq 5$

# Syntax and Semantics of Object Attributes

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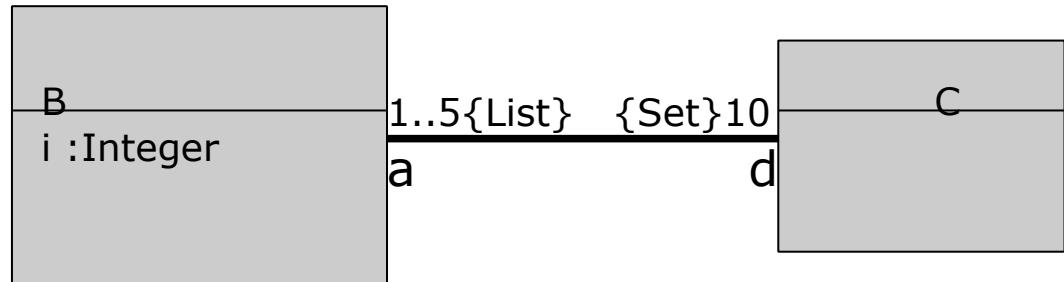
- ❑ Accessor functions are defined as follows for the case of NULL:



- $\text{NULL}.d = \{\}$  -- mapping to the neutral element
- $\text{NULL}.a = []$  -- mapping to the neural element.

# Syntax and Semantics of Object Attributes

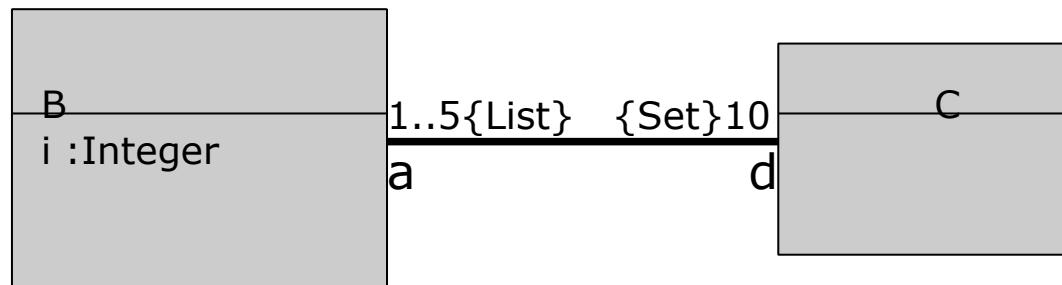
- Cardinalities in Associations can be translated canonically into MOCL invariants:



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# Syntax and Semantics of Object Attributes

- The corresponding association integrity constraints for the \*-\* -case are:



- definition  $\text{ass}_{B.d.a} \equiv \forall x \in B. x \in x.d.a$
- definition  $\text{ass}_{C.a.d} \equiv \forall x \in C. x \in x.a.d$

# Summary

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- ❑ MOAL makes the UML to a real, formal specification language
- ❑ MOAL can be used to annotate Class Models, Sequence Diagrams and State Machines
- ❑ Working out, making explicit the constraints of these Diagrams is an important technique in the transition from Analysis documents to Designs.