

Interactive Theorem Proving and Applications

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TP 1 - Introduction to Isabelle/HOL

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Exercise 1 (Installation)

Install Isabelle(version 2020) from the Isabelle website <https://isabelle.in.tum.de>. Start Isabelle and make yourself familiar with the documentation, in particular the overview “What’s in Main” where “Main” is the standard HOL library we base our exercises on.

Exercise 2 (Editing, Type-Checking, Searching)

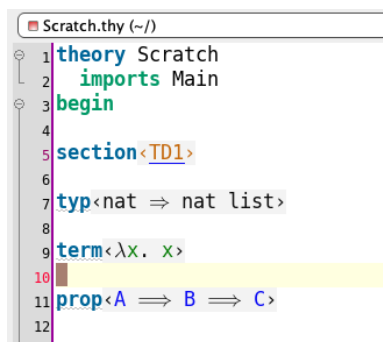
Use the Isabelle commands , `typ`, `term` and `prop` to enter types, terms and propositions into the system, thus using the parser and type-checker of Isabelle.

Start an anonymous session with `isabelle jedit` and create an anonymous session with

```
theory Scratch
  imports Main
begin
```

Questions

1. Enter, parse and type-check (if possible) some types, terms, and propositions. The result should look similar to this : Change types, terms and props at your guise though.



```
Scratch.thy (~/)
1 theory Scratch
2   imports Main
3   begin
4
5   section <TD1>
6
7   typ <nat => nat list>
8
9   term <λx. x>
10
11  prop <A => B => C>
12
```

FIGURE 1 – example caption

2. Enter, parse and type-check (if possible) the term

$$(\lambda x. \lambda y. (\lambda z. (\lambda x. z x) (\lambda y. z y))) (x y)$$

(It might be helpful to add spaces ...) Note how the system represents bound and free variables.

3. Define via a number of **definitions** the Church Numerals of the slides of class 1. The syntax is :

definition *const_name* :: *typ* where "*eqn*"

Which type do Church-Numerals have in the typed λ -calculus?

4. axiomatize the Y-combinator, i.e. enter " $Y f = f(Y f)$ " as axiom into the system. The syntax is :

axiomatization *const_name* :: *typ* where *ax_name* : "*eqn*"

Which (external) type has to be given to the Y-combinator for this axiomatization?

5. use `find_theorems` to browse your theory so far! You will need this possibility later on!
6. Prove that, according to your definitions, *PLUS TWO THREE* is indeed *FIVE*.
Hint : state a **lemma** for this equation, unfold the definitions, and apply the simplification method by `simp`

Exercise 3 (OPTIONAL : Report (IN CASE THAT YOU WANT TO HAVE IT GRADED :))

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max.