Interactive Theorem Proving and Applications

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TP 2 - Datatypes and Induction in Isabelle/HOL

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Exercice 1 (Simple Logical Backward - Proofs)
State the following properties as `lemma` and prove them if possible (give counter-example in this case) :

1. $A \wedge B \wedge C \to B \wedge A$
2. $(\forall x.A \to B(x)) = (A \to (\forall x.B(x)))$
3. $(\forall x.A(x) \wedge B(x)) = ((\forall x.A(x)) \wedge (\forall x.B(x)))$
4. $(\exists x.A(x) \vee B(x)) = ((\exists x.A(x)) \vee (\exists x.B(x)))$
5. $(\forall x.\exists x.A(x)(y)) \to ((\exists x.A(x)) \vee \exists x.B(x))$
6. $(\exists x.\forall y.A\ x\ y) \to (\forall y.\exists x.A\ x\ y)$
7. $((A \to B) \to A) \to A$ (Pierce Law)

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced automated procedures like `simp` and `auto`.

Hint : search for basic logical rules from the `HOL` theory involving the logical connectives/quantifiers.

Hint : Pierce law is only true in classical logic. Think about safeness of implication introduction and consider alternatives.

Exercice 2 (Simple Backward - Proofs on Sets)
It is possible to view functions of type $'\alpha \Rightarrow bool$ as (typed) sets $'\alpha set$; simply consider these functions as characteristic functions and one can see that these concepts are isomorphic. The HOL library `Main` comes with a theory `Set` that is based on these isomorphism. Explore this Set theory and prove :

1. $(A \cup B) \cup C = A \cup (B \cup C)$
2. $(A \cap B) \cup C \subseteq A \cup C$
3. $(A \cup B) \cap C \supseteq A \cap C$

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced automated procedures like `simp` and `auto`.

Hint : With respect to the Isabelle/HOL syntax you might try LaTeX notation and/or consult "What's in Main" in the Documentation.

Exercice 3 (Simple Backward - Proofs on Equality)
Try to rebuild the equational proof of the lecture :

1. $f\ a\ b = a \rightarrow f(f\ a\ b)\ b = c \rightarrow g\ a = g\ c$

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` (in order to control the basic substitutions), and `subst` at wish.

Hint : With respect to the Isabelle/HOL syntax you might try LaTeX notation and/or consult "What's in Main" in the Documentation.

Hint : It might be necessary to use the `subst` rule of equational logic at hand with appropriate substitutions.

Exercice 4 (OPTIONAL : Report)
(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP's SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max.