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TP 6 - Operational and Denotational Semantics

Semaine du 13 fevrier 2023

Exercice 1

Objective : Defining a denotational and Plotking-style operational semantics (big-step semantics) for a small imperative programming language called IMP.

The former kind of semantics models an interpretation function I that maps the abstract syntax terms of IMP to a *semantic domain* D, i.e. $I :: IMP \to D$. As semantic domain serves the state relation $D = (state \times state)set$. The latter kind of semantics representsin the original language and inductively models the transition relation between states via a transition predicate $\_, \longrightarrow_c \_$ of type $state \times state \Rightarrow bool$.

As concrete states, we use functions from some type representing variable names to Integers ; note that there is already some theory on function updates in the `Main`-library, which helps the task significantly. Consider also what can be found on relations.

The language IMP consists of the following concepts :

1. *boolean expressions bexpr*'s, which are functions from *state* to *bool*,
2. *arithmetic expressions aexpr*', which are functions from *state* to *int*,
3. IMP consists of the constructors :
   - the command $SKIP$ that represents the empty program (no effect) ;
   - the command *assignment* that takes a name and an arithmetic expression (denoted $a := E$) ;
   - the sequence command that enchains two commands (denoted $C_1; C_2$)
   - the conditional command, that consists of a boolean condition and two commands (denoted $IF\ E\ THEN\ C_1\ ELSE\ C_2\ FI$) ;
   - the loop command that consists of a boolean condition and a command, the *body* (denoted $WHILE\ E\ DO\ C_1\ OD$).

Tasks :

1. model IMP as datatype, I as recursive function and $\_, \longrightarrow_c \_$ as inductive(set)-definitions. (You will need the lfp to do this for I.)
2. Prove the equivalence of both semantics.
3. Define a Hoare-triple (denoted ($\vdash \{Pre\}\ IMP\ \{Post\}$: "If the precondition is satisfied on some state, and if the program IMP reaches a successor state, then the postcondition must be satisfied on that state").

Exercice 2 (OPTIONAL : Report)

(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP's SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 5 pages max (except screenshots and other figures).