2021

Cycle Ingénieur – 2ème année
Département Informatique

Verification and Validation

Part IV : White-Box Testing

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Towards Static Specification-based Unit Test

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  - What could be a coverage criterion ?
    (or: adequacy criterion,
      telling that we "tested enough")

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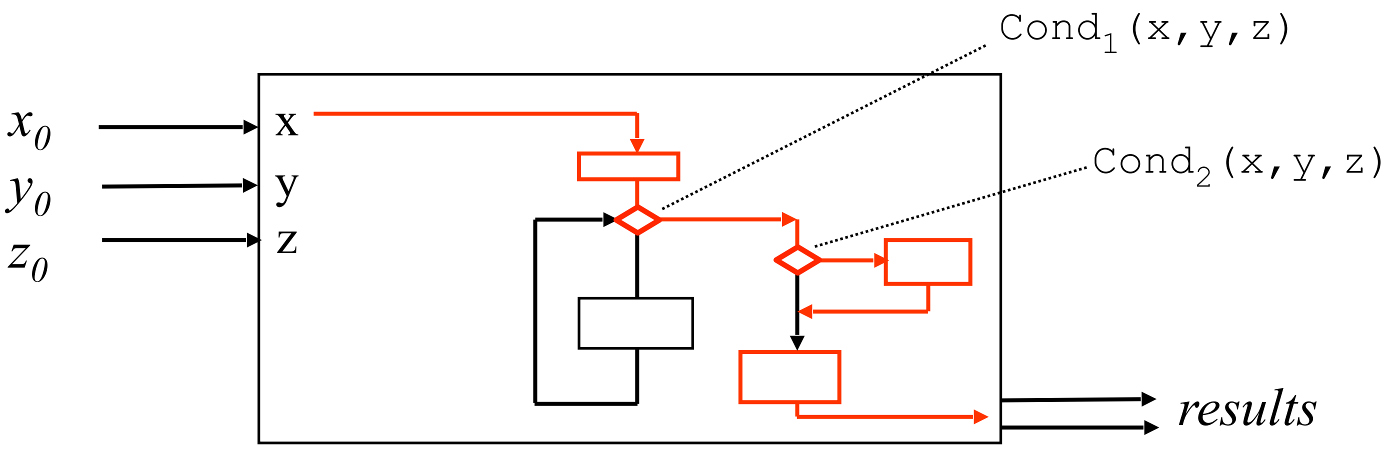
- Lets develop a test method that exploits this !

Static Structural ("white-box") Tests



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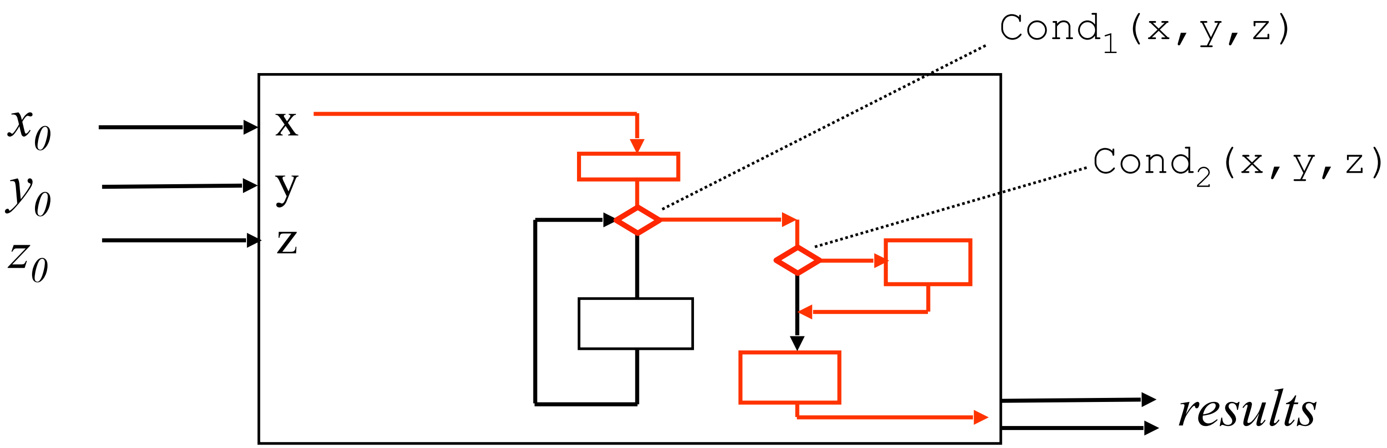

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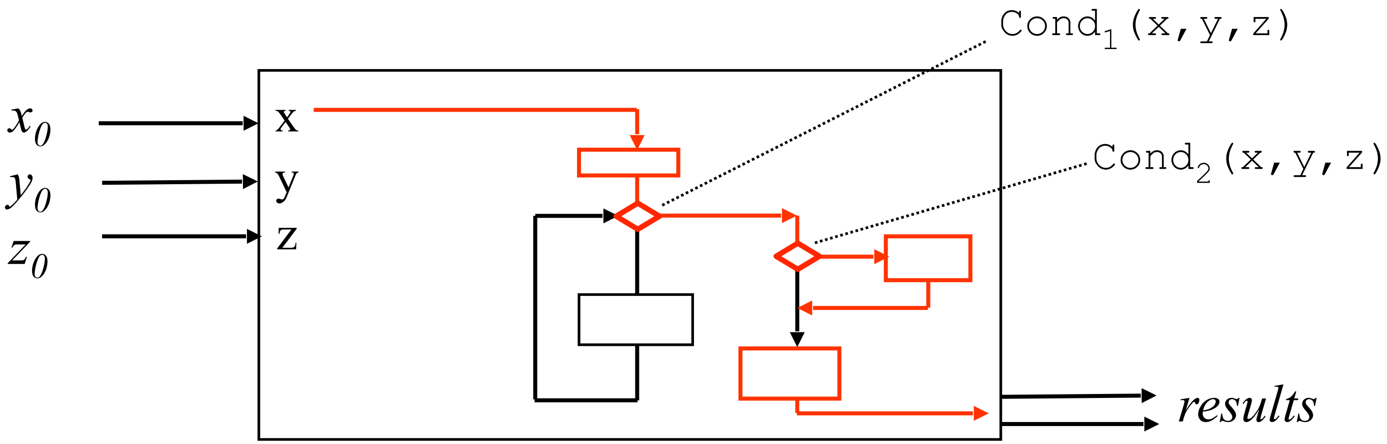

- we select "critical" paths
- specification used to verify the obtained resultants

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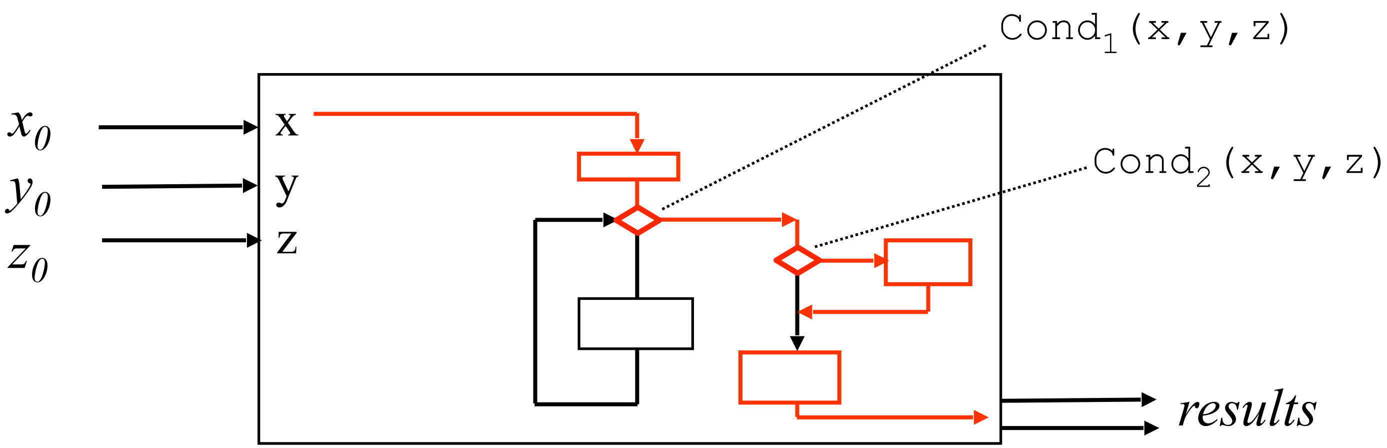

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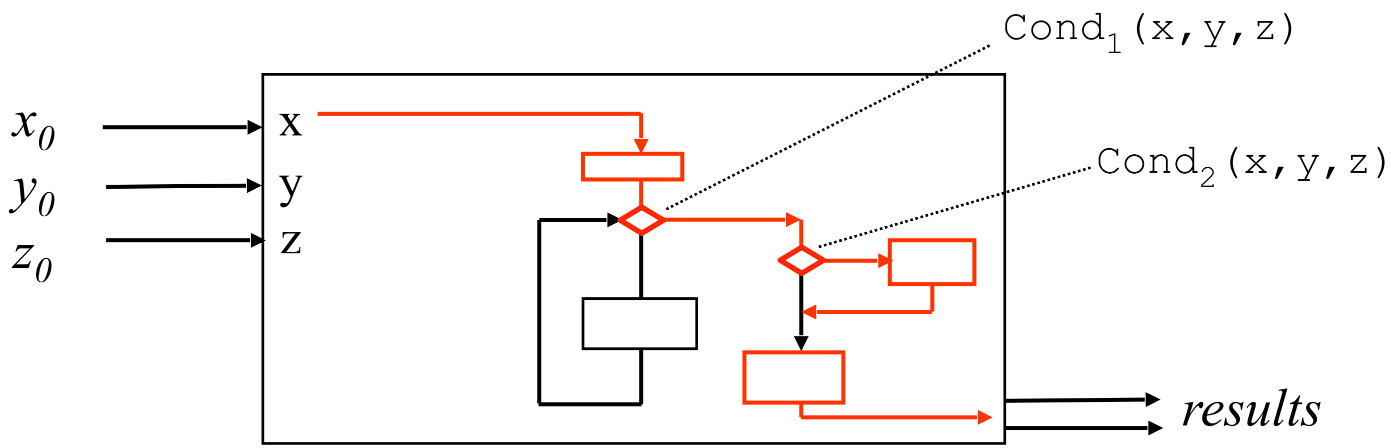

Idea:
a path corresponds to <u>one</u> logical expression over initial values x_0, y_0, z_0 .
corresponding to one test-case (comprising several test data ...)

$$\neg\, Cond_1(x_0,\ y_0,\ z_0) \wedge \neg\, Cond_2(x_0,\ y_0,\ z_0)$$

We are interested either in edges (control flow), or in nodes (data flow)

A Program for the triangle example

```
procedure triangle(j,k,l : positive) is
 eg: natural := 0;
begin
if j + k <= l or k + l  <= j or l + j <= k then
   put(“impossible”);
else if  j = k  then   eg := eg + 1;  end if;
     if  j = l  then   eg := eg + 1;  end if;
     if  l = k  then   eg := eg + 1;  end if;
      if  eg = 0  then  put(“arbitrary”);
     elsif   eg = 1  then put(“isocele”);
     else    put(“equilateral”);
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end if;
end triangle;
```

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- try a certain number of execution "paths" (which ones ? all of them ?)
- find input values to stimulate these paths
- compare the results with expected values (i.e. the specification)

Functional-test vs. structural test?

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Both are complementary and complete each other:

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Both are complementary and complete each other:

- Structural Tests have weaknesses in principle:
  - if you forget a condition, the specification will most likely reveal this !
  - if your algorithm is incomplete, a test on the spec has at least a chance to find this ! (Example: perm generator with 3 loops)

Functional-test vs. structural test?

Both are complementary and complete each other

- Structural Tests have weaknesses in principle:
  for a given specification, there are several possible
  implementations (working more or less differently from the spec):

  - *sorted arrays : linear search ? binary search ?*
  - *$(x, n) \to x^n$: successive multiplication ? quadratic multiplication ?*

  Each implementation demands for different test sets !

Equivalent programs ...

Program 1 :

```
S:=1; P:=N;
while P >= 1 loop S:= S*X; P:= P-1; end loop;
```

Program 2 :

```
S:=1; P:= N;
while P >= 1 loop
   if P mod 2 /= 0 then P := P -1; S := S*X; end if;
   S:= S*S; P := P div 2;
end loop;
```

Both programs satisfy the same spec but ...

- one is more efficient, but more difficult to test.
- test sets for one are not necessarily “good” for the other, too !

Control Flow Graphs

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- assignments
- update operations (on arrays, ..., not discussed here)
- procedure calls (not discussed here !!!)

- conditions and expressions are assumed to be side-effect free

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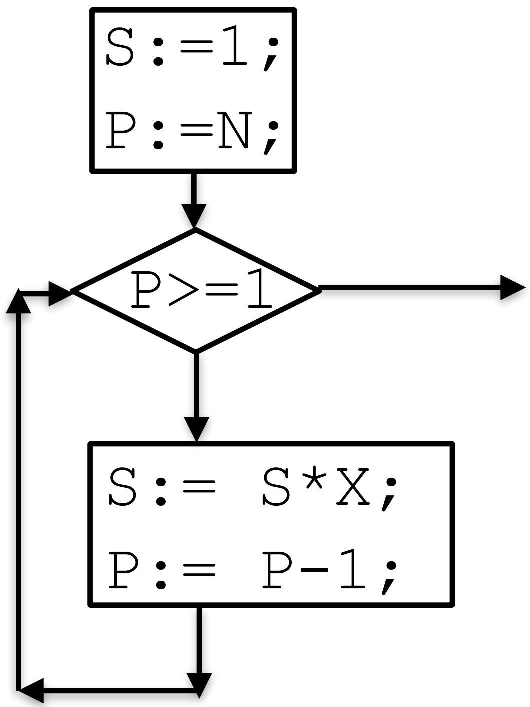


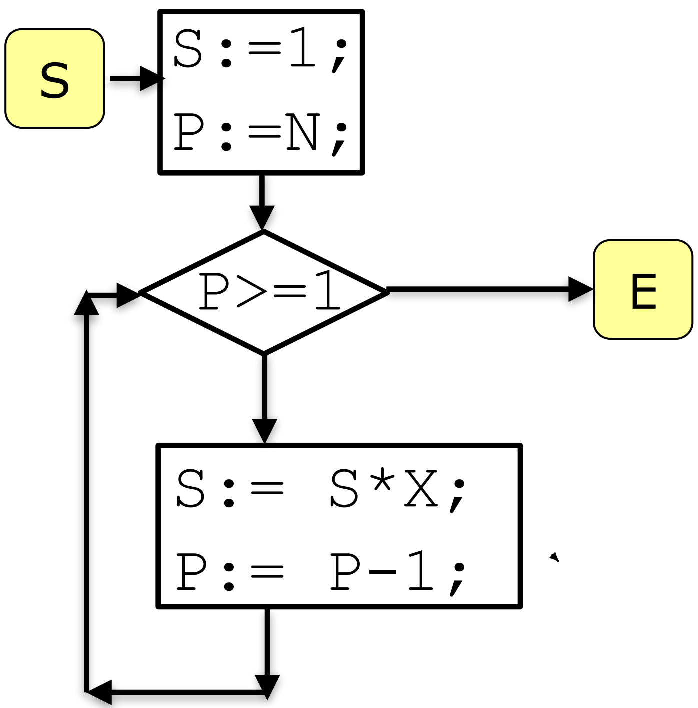
Computing Control Flow Graphs

- Identify longest sequences of assignments
- Erase if_then_elses by branching
- Erase while_loops by loops
- Add entry node and exit loop-arc, entry-arc, exit-arc

A Control-Flow-Graph (CFG) is usually a by-product of a compiler ...

- Example:

  Add entry node and exit loop-arc, entry-arc, exit-arc



Q: What is the CFG

of the body of triangle ?

Revisiting our triangle example ...

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      if  eg = 0  then  put("quelconque");
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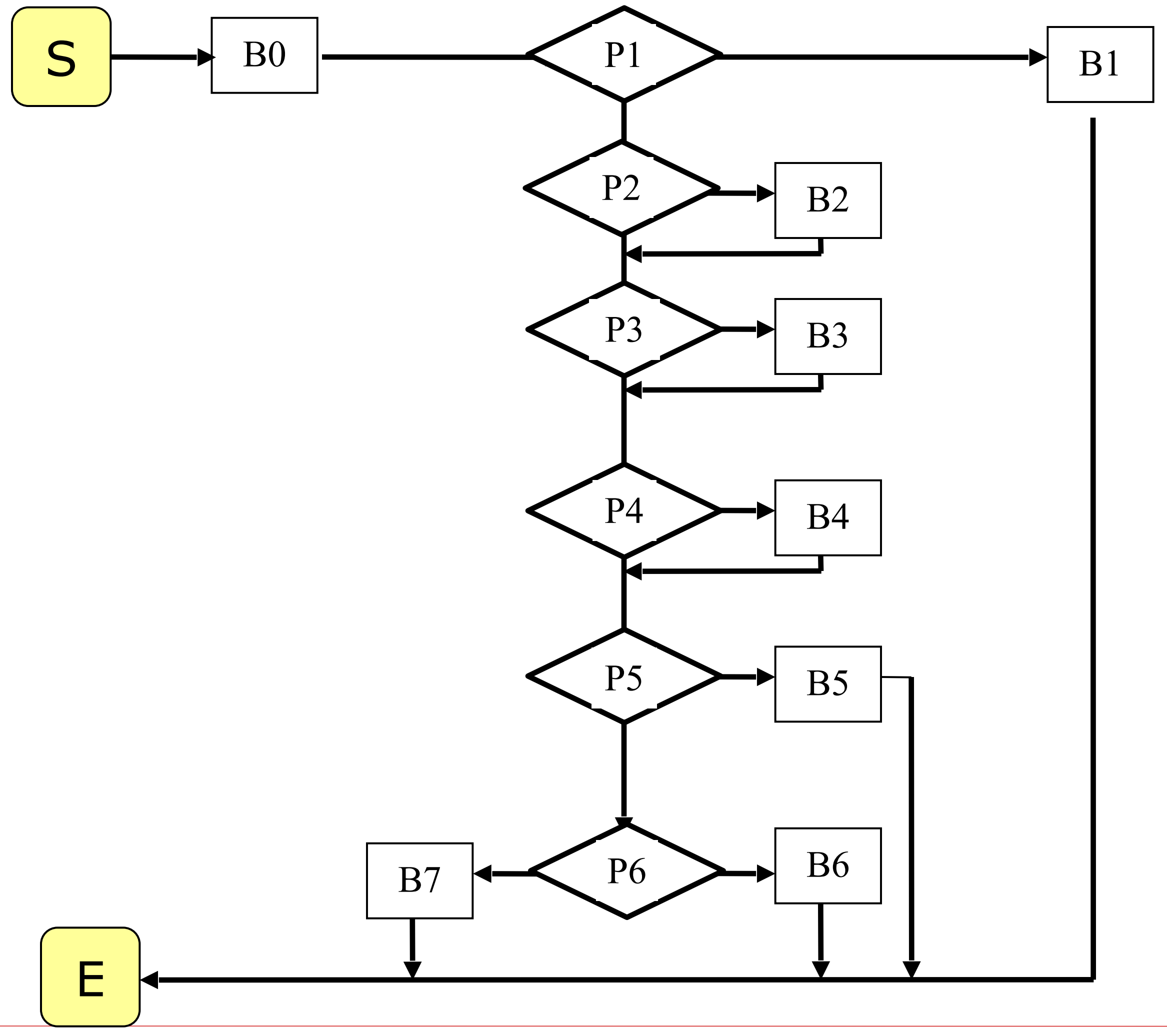
The non-structured control-flow graph of a program



A procedure with loop and return

```
procedure supprime (T: in out Table; p: in out integer;
                          x: in integer) is
 i: integer := 1;
begin
   while  i <> p  loop
      if  T[i].val <> x then  i := i + 1;
      elsif  i = p - 1  then  p := p - 1; return;
       else   T[i] := T[p-1]; p := p -1;  return;
      end if;
   end loop;
end supprime;
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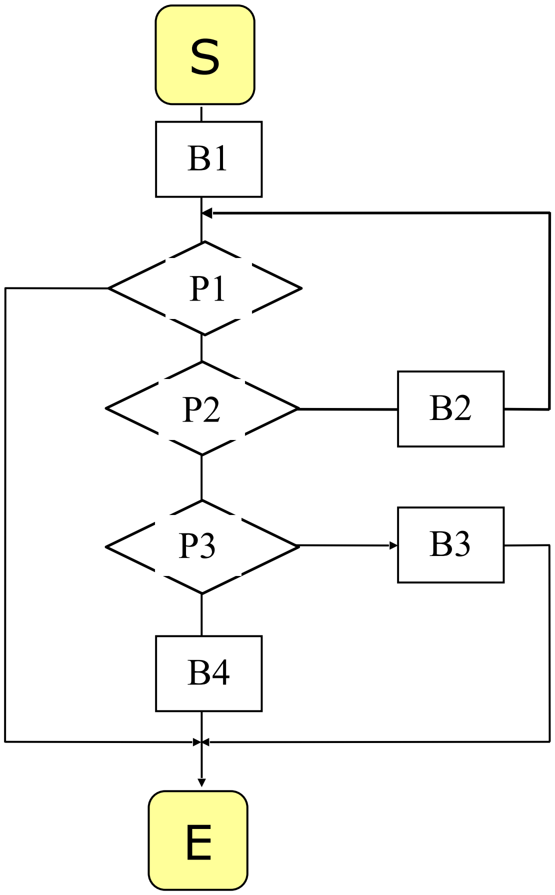

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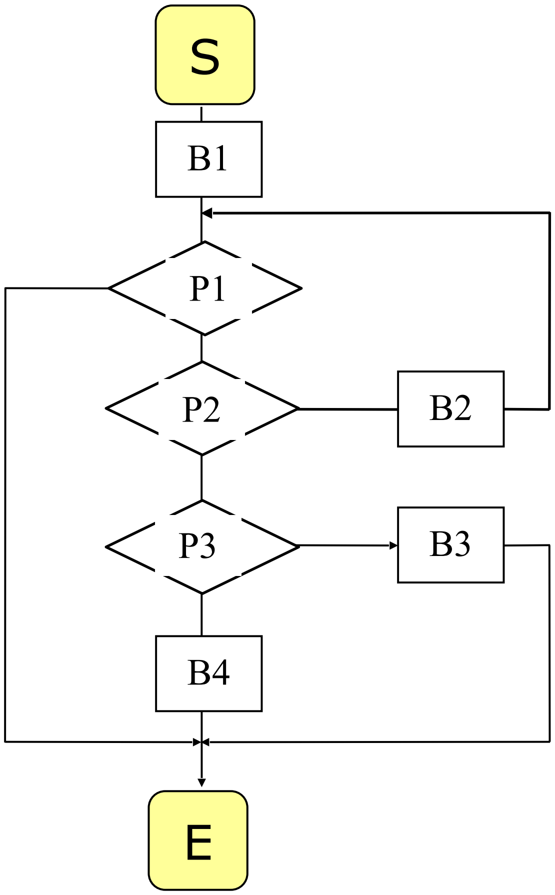

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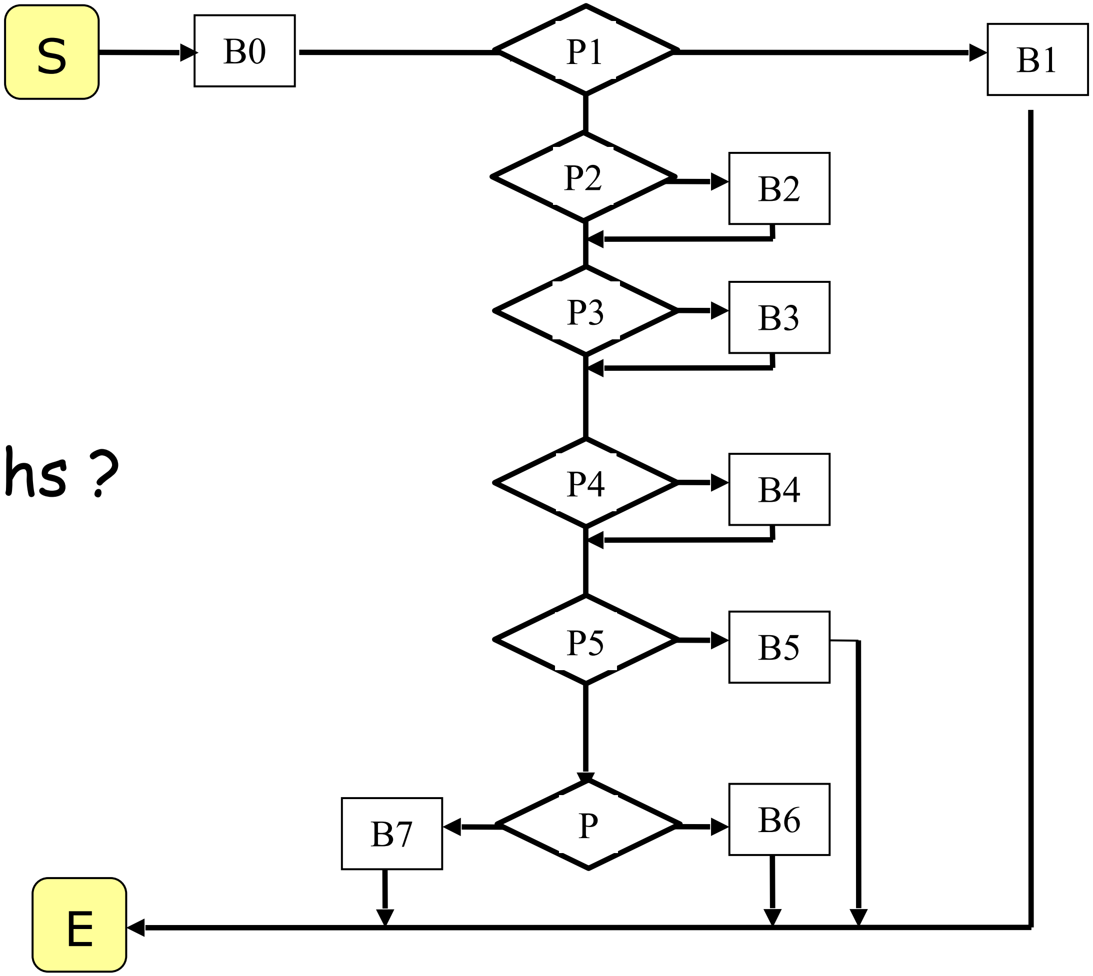

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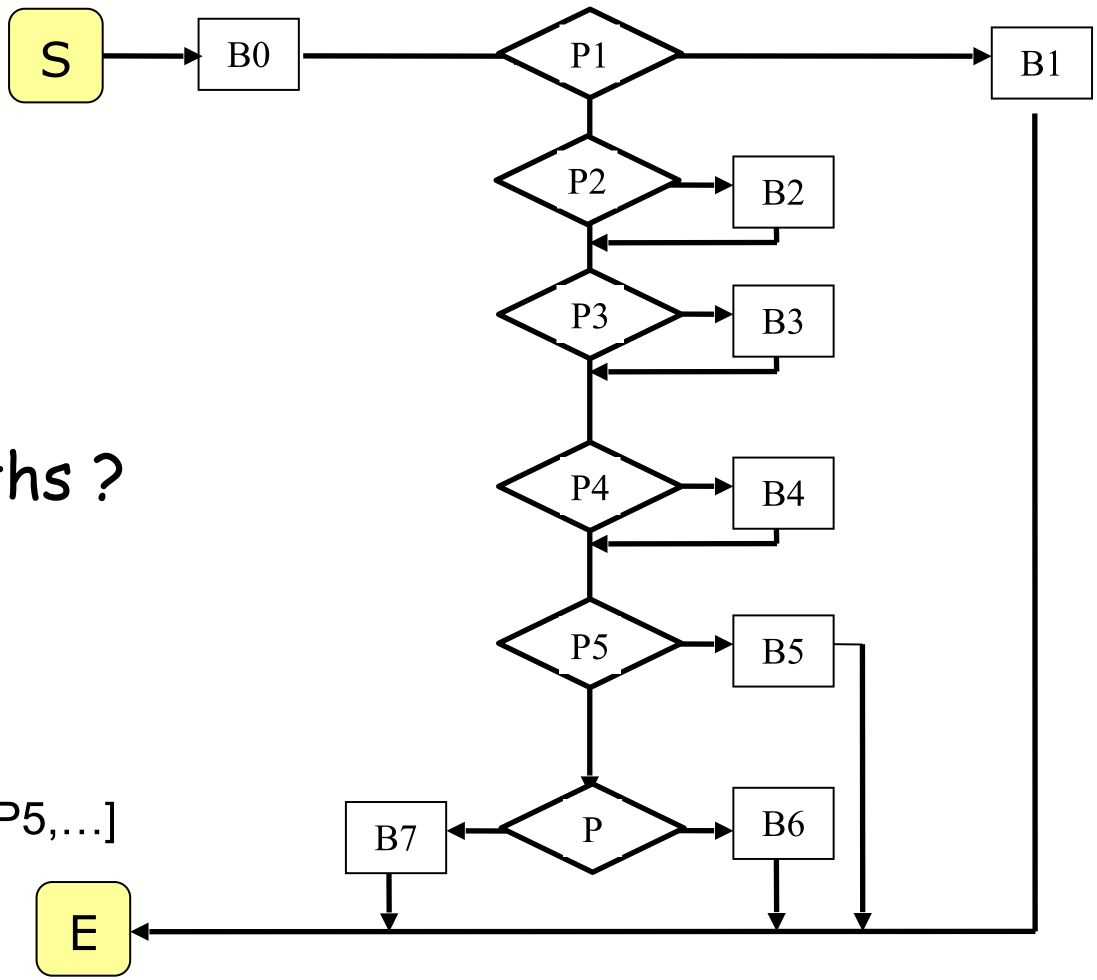

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Consider:

[S,B0,P1,P2,B2,P3,B3,P4,P5,…]

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    i.e. the path condition Φ_M is satisfiable

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The $\underline{x1},\ldots,\underline{xn}$ are the symbolic values for the program variables

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  A state is a table (or: function) that maps a variable *V* to some value of a domain *D*.

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$$\sigma = V \to D$$

- As usual, we denote finite functions as follows:

$$\{ x \mapsto 1,\ y \mapsto 5,\ x \mapsto 12 \}$$

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- For example, if we know that

  $$x \in \{1..10\}$$

  and we have an assignment $x := x+2$, we know:

  $x \in \{3..12\}$ afterwards.

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- This gives rise to the notion of a symbolic state.

$$\sigma_{sym} = V \rightarrow Set(D)$$

We denote the set of possible values by a predicate over the initial state, so:

$$x \mapsto (1 \leq x_0 \wedge x_0 \leq 10)$$

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- thus, after x:= x+2, we know:

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$$= \; 1 + 2 * x_0$$

- An initial symbolic state is a map of the form:

$$\{ x \mapsto x_0, y \mapsto y_0, z \mapsto z_0 \}$$

Basic Blocks *as* Substitutions

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Symbolic Pre-State σ_{sym}

$x \mapsto x_0$
$y \mapsto y_0+3*x_0$
$z \mapsto z_0$
$i \mapsto i_0$

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Symbolic Post-State σ'_{sym}

x	↦ x_0
y	↦ y_0+3*x_0
z	↦ $z_0+y_0+4*x_0+1$
i	↦ $y_0+ 4*x_0+1$

Basic Blocks as Substitutions

Symbolic Pre-State σ_{sym}	Block	Symbolic Post-State σ'_{sym}
$x \mapsto x_0$	i := x+y+1	$x \mapsto x_0$
$y \mapsto y_0+3*x_0$	z := z+i	$y \mapsto y_0+3*x_0$
$z \mapsto z_0$		$z \mapsto z_0+y_0+4*x_0+1$
$i \mapsto i_0$		$i \mapsto y_0+4*x_0+1$

x_0, y_0 and z_0 represent the initial values of x, y et z.

i is supposed to be a un-initialized local variable.

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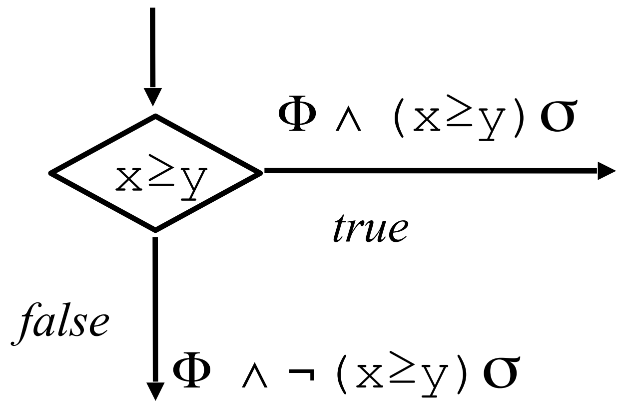
x_0, y_0 and z_0 represent the initial values of x, y et z.

i is supposed to be a un-initialized local variable.

Thus, we update the symbolic state whenever we pass a basic block on our path.

Symbolic Execution

σ_{sym}



Thus, we update the path-condition whenever we pass a decision node on our path.

Example: A Symbolic Path Execution

Recall

```
procedure supprime (T: in out Table; p: in out integer;
                    x: in integer) is
 i: integer := 1;
begin
   while  i <> p  loop
      if  T[i] <> x then  i := i + 1;
      elsif  i = p - 1 then  p := p - 1; return;
       else   T[i] := T[p-1];     p := p - 1; return;
      end if;
   end loop;
end supprime;
```

Example: A Symbolic Path Execution

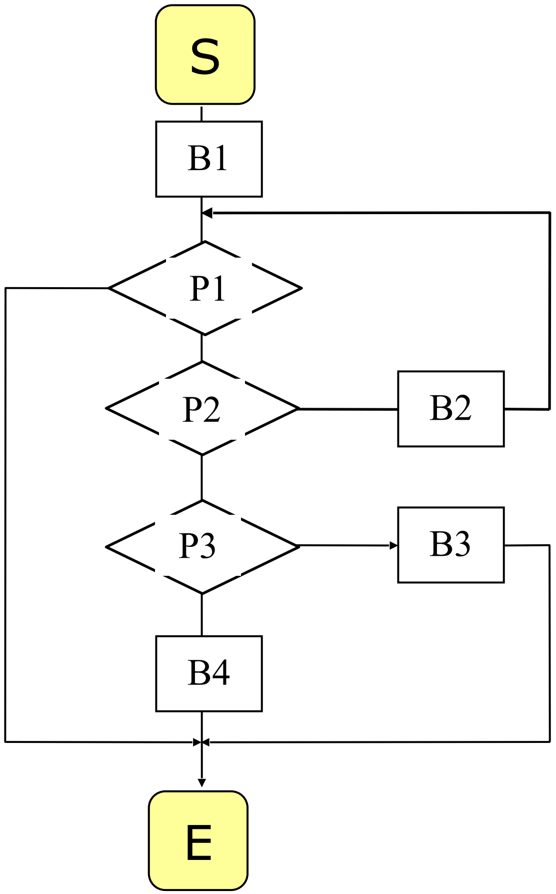
… and the corresponding
control flow graph.

We want to execute the path:

[S,B1,P1,E]

Example: A Symbolic Path Execution

… and the corresponding control flow graph.



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$\Phi \mapsto$ `True`

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, E]

$\Phi \mapsto \mathtt{True}$

T	$\mapsto$	T_0
p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	i_0

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, E]

$\Phi \mapsto$ True $\quad\quad \Phi \mapsto$ True

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Example: A Symbolic Path Execution

We want to execute the path:

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$\Phi \mapsto$ True

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p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	1

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, E]

S: $\Phi \mapsto \texttt{True}$

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p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	i_0

B1: $\Phi \mapsto \texttt{True}$

T	$\mapsto$	T_0
p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	1

P1: $\Phi \mapsto \neg(\texttt{i<>p})\sigma_{B1}$

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, E]

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p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	i_0

B1: $\Phi \mapsto \texttt{True}$

T	$\mapsto$	T_0
p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	1

P1: $\Phi \mapsto \neg (i <> p)\sigma_{B1}$

T	$\mapsto$	T_0
p	$\mapsto$	p_0
x	$\mapsto$	X_0
i	$\mapsto$	1

Example: A Symbolic Path Execution

We want to execute the path:

[S,	B1,	P1,	E]
$\Phi \mapsto \mathtt{True}$	$\Phi \mapsto \mathtt{True}$	$\Phi \mapsto \neg\,(\mathtt{i<>p})\,\sigma_{B1}$	$\Phi \mapsto 1 = p_0$
$\mathtt{T} \mapsto \mathtt{T}_0$	$\mathtt{T} \mapsto \mathtt{T}_0$	$\mathtt{T} \mapsto \mathtt{T}_0$	
$\mathtt{p} \mapsto \mathtt{p}_0$	$\mathtt{p} \mapsto \mathtt{p}_0$	$\mathtt{p} \mapsto \mathtt{p}_0$	
$\mathtt{x} \mapsto \mathtt{X}_0$	$\mathtt{x} \mapsto \mathtt{X}_0$	$\mathtt{x} \mapsto \mathtt{X}_0$	
$\mathtt{i} \mapsto \mathtt{i}_0$	$\mathtt{i} \mapsto 1$	$\mathtt{i} \mapsto 1$	

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, E]

S	B1	P1	E
$\Phi \mapsto$ True	$\Phi \mapsto$ True	$\Phi \mapsto \neg(\mathtt{i<>p})\,\sigma_{B1}$	$\Phi \mapsto 1 = p_0$
$T \mapsto T_0$	$T \mapsto T_0$	$T \mapsto T_0$	$T \mapsto T_0$
$p \mapsto p_0$	$p \mapsto p_0$	$p \mapsto p_0$	$p \mapsto p_0$
$x \mapsto X_0$	$x \mapsto X_0$	$x \mapsto X_0$	$x \mapsto X_0$
$i \mapsto i_0$	$i \mapsto 1$	$i \mapsto 1$	$i \mapsto 1$

Example: A Symbolic Path Execution

Result:

Test-Case:

For the path M=[S,B1,P1,E]

we have the path condition $\Phi \mapsto p_0 = 1$

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Example: A Symbolic Path Execution

Result:

Test-Case:

For the path M=[S,B1,P1,E]

we have the path condition $\Phi \mapsto$ `p`$_0$ `= 1`

A concrete Test,
satisfying Φ

`T`	$\mapsto$	`mtTab`
`p`	$\mapsto$	`1`
`x`	$\mapsto$	`17`

Example: A Symbolic Path Execution

... and the corresponding

control flow graph.

We want to execute the path:

[S,B1,P1,P2,B2,P1,E]

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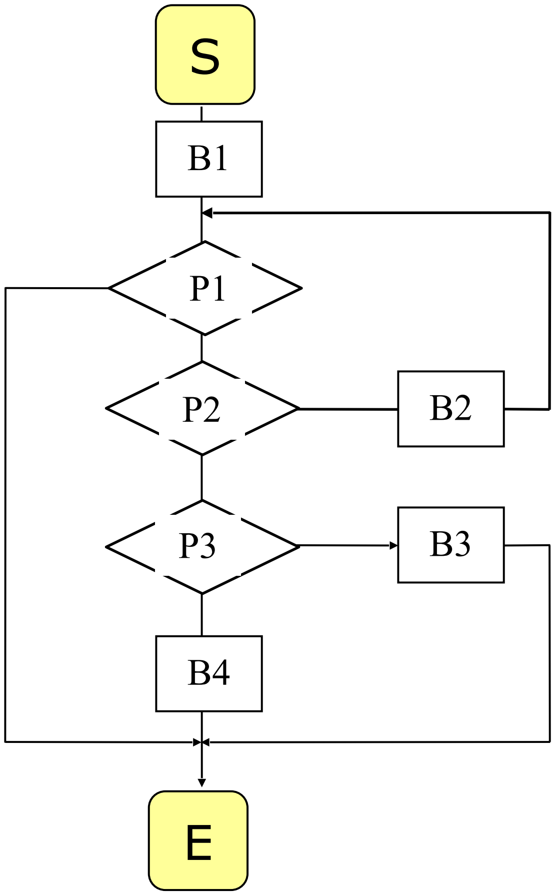

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$T \mapsto T_0$	T_0					
$p \mapsto p_0$	p_0					
$x \mapsto x_0$	x_0					
$i \mapsto i_0$	1					

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, P2, B2, P1, E]

$\Phi \mapsto$ True	True	(i<>p) σ_{B1}				
$T \mapsto T_0$	T_0					
$p \mapsto p_0$	p_0					
$x \mapsto x_0$	x_0					
$i \mapsto i_0$	1					

Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, P2, B2, P1, E]

$\Phi \mapsto$ True	True	$(i<>p)\sigma_{B1} \equiv p_0 \neq 1$				
$T \mapsto T_0$	T_0					
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$x \mapsto x_0$	x_0	x_0				
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We want to execute the path:

[S, B1, P1, P2, B2, P1, E]

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$T \mapsto T_0$	T_0	T_0				
$p \mapsto p_0$	p_0	p_0				
$x \mapsto x_0$	x_0	x_0				
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Example: A Symbolic Path Execution

We want to execute the path:

[S, B1, P1, P2, B2, P1, E]

S	B1	P1	P2	B2	P1	E
$\Phi \mapsto$ True	True	$(i <> p)\sigma_{B1} \equiv p_0 \neq 1$	$p_0 \neq 1 \land (T[i] \neq x)\sigma_{B1}$			
$T \mapsto T_0$	T_0	T_0				
$p \mapsto p_0$	p_0	p_0				
$x \mapsto x_0$	x_0	x_0				
$i \mapsto i_0$	1	1				

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$T \mapsto T_0$	T_0	T_0	T_0			
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$T \mapsto T_0$	T_0	T_0	T_0	T_0		
$p \mapsto p_0$	p_0	p_0	p_0	p_0		
$x \mapsto x_0$	x_0	x_0	x_0	x_0		
$i \mapsto i_0$	1	1	1	$(i+1)\sigma_{B1}$		

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$T \mapsto T_0$	T_0	T_0	T_0	T_0		
$p \mapsto p_0$	p_0	p_0	p_0	p_0		
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$T \mapsto T_0$	T_0	T_0	T_0	T_0	T_0	
$p \mapsto p_0$	p_0	p_0	p_0	p_0	p_0	
$x \mapsto x_0$	x_0	x_0	x_0	x_0	x_0	
$i \mapsto i_0$	1	1	1	$(i+1)\sigma_{B1}$	2	

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$T \mapsto T_0$	T_0	T_0	T_0	T_0	T_0	
$p \mapsto p_0$	p_0	p_0	p_0	p_0	p_0	
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$T \mapsto T_0$	T_0	T_0	T_0	T_0	T_0	T_0
$p \mapsto p_0$	p_0	p_0	p_0	p_0	p_0	p_0
$x \mapsto x_0$	x_0	x_0	x_0	x_0	x_0	x_0
$i \mapsto i_0$	1	1	1	$(i+1)\sigma_{B1}$	2	2

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$i \mapsto i_0$	1	1	1	$(i+1)\sigma_{B1}$	2	2

Example: A Symbolic Path Execution

Result: Test-Case for Path

M = [S,B1,P1,P2,B2,P1,E]

Path Condition: $\Phi := T_0[1] \neq X_0 \wedge p_0 = 2$

Example: A Symbolic Path Execution

Result: Test-Case for Path

M = [S,B1,P1,P2,B2,P1,E]

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A concrete Test,
satisfying Φ

Example: A Symbolic Path Execution

Result: Test-Case for Path

$$M = [S,B1,P1,P2,B2,P1,E]$$

Path Condition: $\Phi := T_0[1] \neq X_0 \wedge p_0 = 2$

A concrete Test, satisfying Φ

$T \mapsto [3]$
$p \mapsto 2$
$x \mapsto 17$

Paths and Test Sets

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In (this version of) program-based testing

a test case with a (feasable) path

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  - = a collection of values for variables (params and global)
    - (+ the output values described by the specification)

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Paths and Test Sets

In (this version of) program-based testing
a test case with a (feasable) path

- a test case ≈ a path M in the CFG
  = a collection of values for variables (params and global)
  (+ the output values described by the specification)

- a test case set ≈ a finite set of paths of the CFG
  = a finite set of input values and
  a set of expected outputs.

Unfeasible paths and decidability

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- In general, it is undecidable of a path is feasible ...

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- In general, a first-order formula over arithmetic is undecidable ...
- **... Indecidable = it is known (mathematically proven) that there is no algorithm; this is worse than "we know none" !~**

BUT: for many relevant programs, practically good solutions exist (Z3, Simplify, CVC4, AltErgo ...)

A Challenge-Example (The Collatz-Function):

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... **A HAIRY EXAMPLE:**

```
while x <> 1 loop
    if pair(x) then x := x / 2;
    else x := 3 * x + 1;
    end if;
end loop;
```

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while x <> 1 loop
    if pair(x) then x := x / 2;
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- does this function terminate for all x ?
- or equivalently: is ***end loop*** reached for all x ?

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ANSWER : unknown

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... **A HAIRY EXAMPLE:**

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while x <> 1 loop
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end loop;
```

- this implies that we can not always know that infeasible paths exist !

The Triangle Prog without Unfeasible Paths

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```
procedure triangle(j,k,l)
```

The Triangle Prog without Unfeasible Paths

```
procedure triangle(j,k,l)
begin
```

The Triangle Prog without Unfeasible Paths

```
procedure triangle(j,k,l)
begin
  if j k<=l or k+l<=j or l+j<=k  then  put("impossible");
```

The Triangle Prog without Unfeasible Paths

```
procedure triangle(j,k,l)
begin
  if j k<=l or k+l<=j or l+j<=k  then  put("impossible");
  elsif j = k and k = l  then put("equilateral");
```

The Triangle Prog without Unfeasible Paths

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procedure triangle(j,k,l)
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  else   put("quelconque");
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The Triangle Prog without Unfeasible Paths

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procedure triangle(j,k,l)
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end if;
```

The Triangle Prog without Unfeasible Paths

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  else   put("quelconque");
end if;
end;
```

- In the contrary, there are programs where all paths are feasible
- That is rare, however.
- Worse: in practice the probability for a path to be feasible is smaller the longer the path gets.

The notion of a "coverage criterion"

The notion of a "coverage criterion"

A coverage criterion is a function mapping a CFG to a particular subset of its paths …

- the set of paths covering all basic blocks
- the set of paths covering all instructions
- the set with all loops are traversed
- a particular subset of calls/labels occurring in the CFG has been covered
- …

Well-known Coverage Criteria I

Criterion C = AllInstructions(CFG):

For all nodes N in CFG (basic instructions or decisions) exists a path in C that contains N

Well-known Coverage Criteria II

Criterion C = AllTransitions(CFG):

For all arcs A in the CFG exists a path in C that uses A

Well-known Coverage Criteria III

Criterion C = AllPaths(CFG):

All possible paths ...

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☹ Whenever there is a loop, C is infinite !

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Criterion C = AllPaths(CFG):

All possible paths ...

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- ☞ weaker variant: $\text{AllPaths}_k(\text{CFG})$.

  We limit the paths through a loop to maximally k times …

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A Hierarchy of Coverage Criteria

- $\text{AllPaths(CFG)} \supseteq$

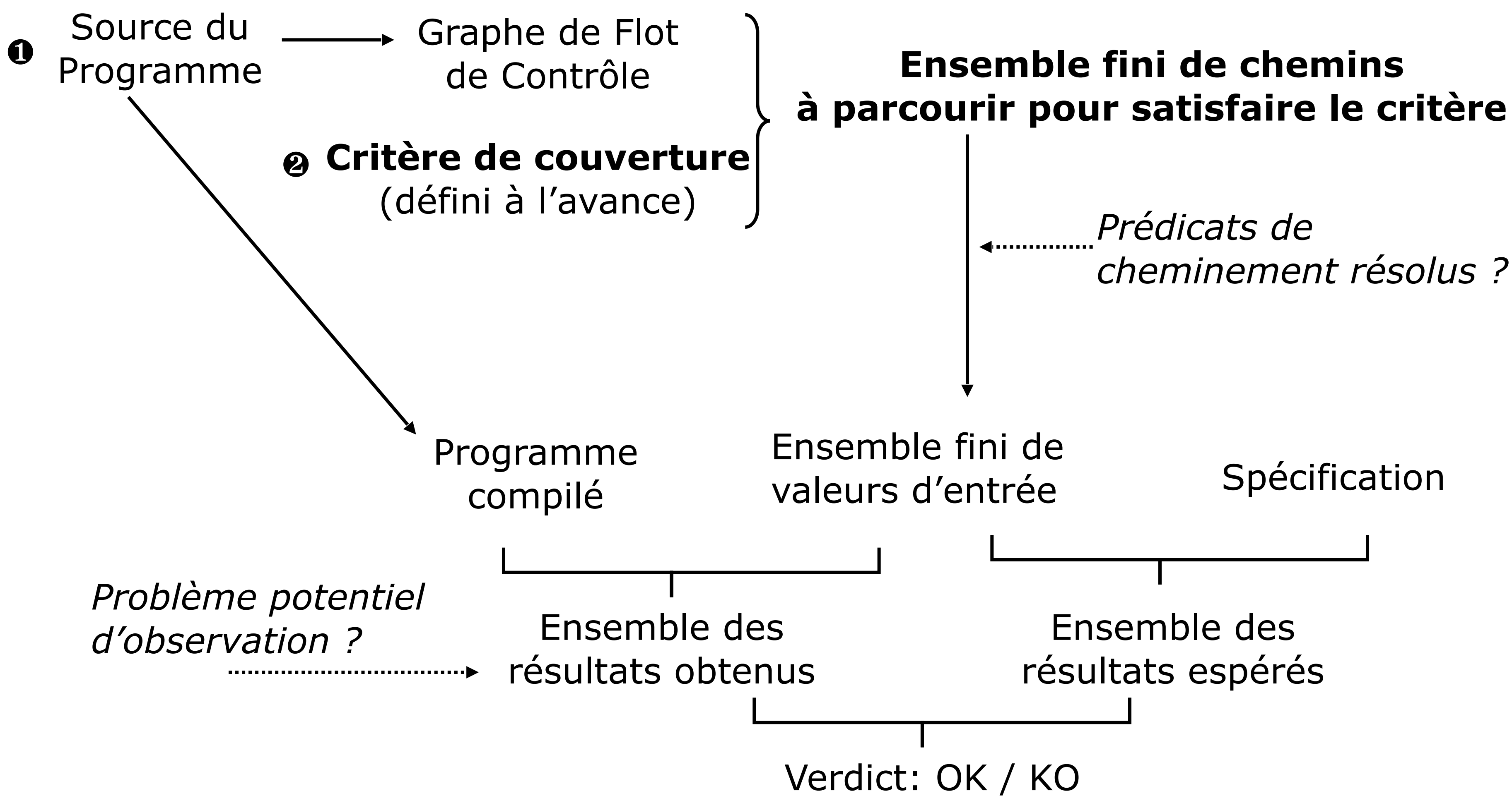
  $\text{AllPaths}_k\text{(CFG)} \supseteq$

  $\text{AllTransitions(CFG)} \supseteq$

  $\text{AllInstructions(CFG)}$

- Each of these implications reflects a proper containment; the other way round is never true.

Using Coverage Criteria 1



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- ... used in tools like JavaPathFinder-SE or Pex
- Core-Concept: Feasible Paths in a Control Flow Graph
- Although many theoretical negative results on key properties, good practical approximations are available
- CFG based Coverage Critieria give rise to a hierarchy

Schmankerle

- Program:

```
int f (int a) {
    int i = 0;
    int tm = 1;
    int sum = 1;
    while(sum <= a) {
        i = i+1;
        tm = tm+2;
        sum = tm+sum;
    }
    return i;
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Specification:

pre : $a \geq 0$

post: $a \leq result^2 \wedge a < (result+1)^2$

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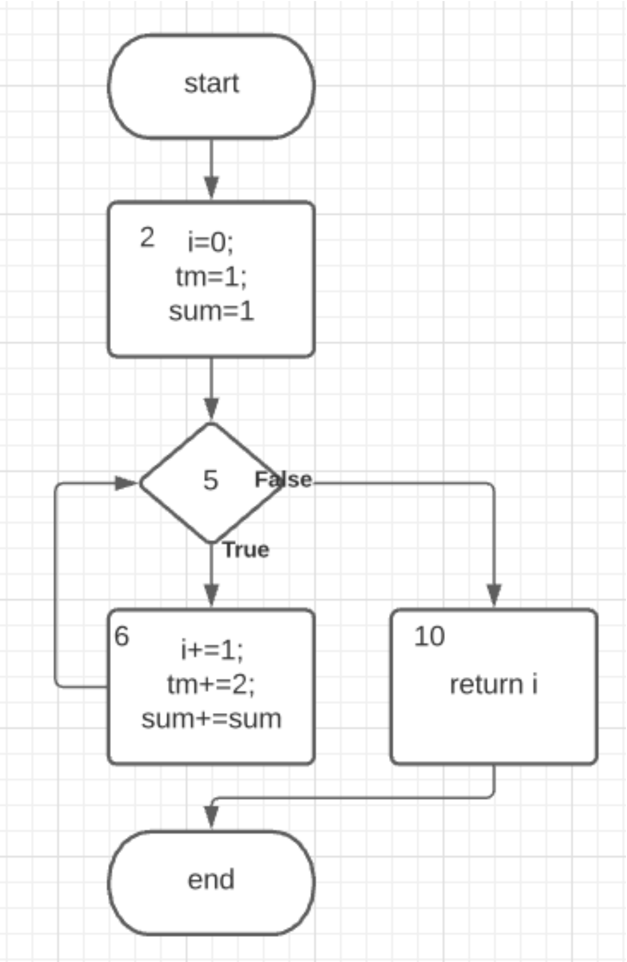

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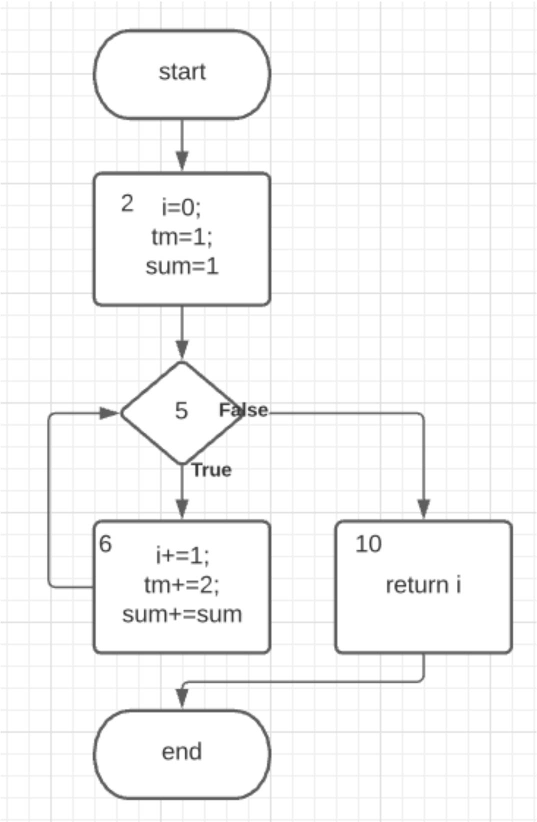

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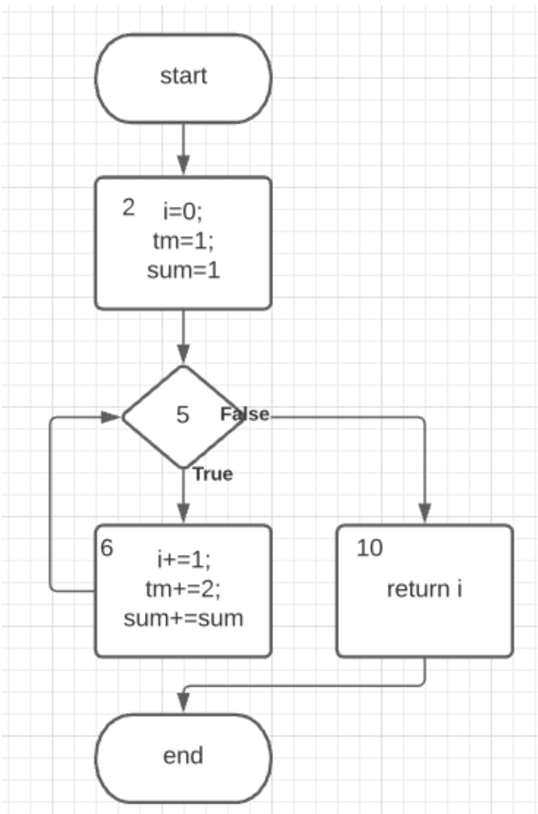

- For example:

AllInstructions(CFG)={[start,2,5,6,5,10,end]}

AllTransitions(CFG)={[start,2,5,6,5,10,end]}

AllPath_3(CFG)={[start,2,5,10,end],
[start,2,5,6,5,10,end],
[start,2,5,6,6,5,10,end],
[start,2,5,6,6,6,5,10,end]}

AllPath(CFG)={ $k \in \mathbb{N}$ |
[start,2,5,$(6)^k$,5,10,end]}
(infinite !)

Example: A Symbolic Path Execution

We want to execute the path from $\mathrm{AllPath}_3$:

[S, 2, 5, 6, 5, 10, E]

`res = 1`

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$\Phi \mapsto a_0 \geq 0$	$a_0 \geq 0$					res = 1
a $\mapsto$ a_0	a_0					
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Example: A Symbolic Path Execution

We want to execute the path from AllPath₃:

[S, 2, 5, 6, 5, 10, E]

$\Phi \mapsto a_0 \geq 0$	$a_0 \geq 0$	$(sum \leq a)\sigma_2 \wedge a_0 \geq 0$	$1 \leq a_0 \wedge a_0 \geq 0$	$1 \leq a_0 \wedge \neg(sum \leq a)\sigma_6$	$1 \leq a_0 \wedge 4 > a_0$	$1 \leq a_0 \wedge 4 > a_0 \wedge res = 1$
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Example: A Symbolic Path Execution

Result:

Test-Case:

For the path M=[start,2,5,6,5,10,end]

we have the path condition $\Phi \mapsto 1 \leq a_0 \wedge 4 > a_0$

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Execution of program with this test vector 3: f(3) = 1

Verification of the post-condition: post(3, 1) = true