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<https://www.lri.fr/~wolff/teach-material/2023-2024/M2-CSMR/index.html>

TP 2 - Datatypes and Induction in Isabelle/HOL

Semaine du 11 janvier 2021

Exercice 1 (Simple Logical Backward - Proofs)

State the following properties as `lemma` and prove them if possible (give counter-example in this case) :

1. $A \wedge B \wedge C \rightarrow B \wedge A$
2. $(\forall x. A \rightarrow B(x)) = (A \rightarrow (\forall x. B(x)))$
3. $(\forall x. A(x) \wedge B(x)) = ((\forall x. A(x)) \wedge (\forall x. B(x)))$
4. $(\exists x. A(x) \vee B(x)) = ((\exists x. A(x)) \vee (\exists x. B(x)))$
5. $(\forall x. \exists x. A(x)(y)) \rightarrow ((\exists x. A(x)) \vee \exists x. B(x))$
6. $(\exists x. \forall y. A x y) \rightarrow (\forall y. \exists x. A x y)$
7. $((A \rightarrow B) \rightarrow A) \rightarrow A$ (Pierce Law)

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced automated procedures like `simp` and `auto`.

Hint : search for basic logical rules from the HOL theory involving the logical connectives/quantifiers.

Hint : Pierce law is only true in classical logic. Think about safeness of implication introduction and consider alternatives.

Exercice 2 (Simple Backward - Proofs on Sets)

It is possible to view functions of type $'a \Rightarrow bool$ as (typed) sets $'a set$; simply consider these functions as characteristic functions and one can see that these concepts are isomorphic. The HOL library `Main` comes with a theory `Set` that is based on these isomorphism. Explore this Set theory and prove :

1. $(A \cup B) \cup C = A \cup (B \cup C)$
2. $(A \cap B) \cup C \subseteq A \cup C$
3. $(A \cup B) \cap C \supseteq A \cap C$

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced automated procedures like `simp` and `auto`.

Hint : With respect to the Isabelle/HOL syntax you might try LaTeX notation and/or consult "What's in Main" in the Documentation.

Exercice 3 (Simple Backward - Proofs on Equality)

Try to rebuild the equational proof of the lecture :

1. $f a b = a \rightarrow f(f a b) b = c \rightarrow g a = g c$

Objective : try to solve these proofs with elementary Isabelle proof methods, i.e. `rule`, `rule_tac`, `erule`, `erule_tac` (in order to control the basic substitutions), and `subst` at wish.

Hint : With respect to the Isabelle/HOL syntax you might try LaTeX notation and/or consult "What's in Main" in the Documentation.

Hint : It might be necessary to use the `subst` rule of equational logic at hand with appropriate substitutions.

Exercice 4 (OPTIONAL : Report)

(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP'S SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max.